The Geometry of Causality

Multi-token Geometry of Interaction and Its Causal Unfolding

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I. Context and Contributions

Operational and Denotational Semantics

$$\frac{M \Downarrow 2 \qquad N \Downarrow 6}{M + N \Downarrow 8}$$

Operational and Denotational Semantics

$$\frac{M \Downarrow 2 \qquad N \Downarrow 6}{M + N \Downarrow 8} \qquad \frac{M \leadsto M'}{M + N \leadsto M' + N}$$

Operational and Denotational Semantics

$$\frac{M \Downarrow 2 \qquad N \Downarrow 6}{M + N \Downarrow 8} \qquad\qquad \frac{M \rightsquigarrow M'}{M + N \rightsquigarrow M' + N}$$

$$x + y \rightsquigarrow ?$$

Game Semantics

$$x : \mathbb{N} \quad , \quad y : \mathbb{N} \quad \vdash x + y \quad : \quad \mathbb{N}$$

Game Semantics

$$x : \mathbb{N} \quad , \quad y : \mathbb{N} \quad \vdash x + y \quad : \quad \mathbb{N}$$

$$\mathsf{Q}$$

Game Semantics

$$x : \mathbb{N} \quad , \quad y : \mathbb{N} \quad \vdash x + y \quad : \quad \mathbb{N}$$

Q

Q

Game Semantics

$$x : \mathbb{N} \quad , \quad y : \mathbb{N} \quad \vdash x + y \quad : \quad \mathbb{N}$$

Q

Q

3

Game Semantics

$$x : \mathbb{N} \quad , \quad y : \mathbb{N} \quad \vdash x + y \quad : \quad \mathbb{N}$$

$x : \mathbb{N}$	$y : \mathbb{N}$	$\mathbb{N}$
		Q
Q		
3		
	Q	

Game Semantics

$$x : \mathbb{N} \quad , \quad y : \mathbb{N} \quad \vdash x + y \quad : \quad \mathbb{N}$$

$x : \mathbb{N}$	$y : \mathbb{N}$	$\mathbb{N}$
		Q
Q		
3		
	Q	
	6	

Game Semantics

$x : \mathbb{N}$	,	$y : \mathbb{N}$	$\vdash x + y$	:	$\mathbb{N}$
					Q
Q					
3					
		Q			
		6			
					9

Game Semantics

$$F \;:\; \mathbb{U} \;\to\; \mathbb{N} \;\vdash\; F() \;:\; \mathbb{N}$$

Game Semantics

$$F \quad : \quad \mathbb{U} \quad \rightarrow \quad \mathbb{N} \quad \vdash \quad F() \quad : \quad \mathbb{N}$$

$$\mathrm{Q}$$

Game Semantics

$$F : \mathbb{U} \to \mathbb{N} \vdash F() : \mathbb{N}$$

Q

Q

Game Semantics

$$F : \mathbb{U} \to \mathbb{N} \vdash F() : \mathbb{N}$$

F	:	$\mathbb{U}$	$\to$	$\mathbb{N}$	$\vdash$	$F()$	:	$\mathbb{N}$
								Q
				Q				
				2				

Game Semantics

$$
\begin{array}{ccccccccc}
F & : & \mathbb{U} & \to & \mathbb{N} & \vdash & F() & : & \mathbb{N} \\
 & & & & & & & & \color{red}{Q} \\
 & & & & \color{green}{Q} & & & & \\
 & & & & \color{red}{2} & & & & \\
 & & & & & & & & \color{green}{2}
\end{array}
$$

Game Semantics

$$F : \mathbb{U} \rightarrow \mathbb{N} \vdash F() : \mathbb{N}$$

$$\hphantom{F : \mathbb{U} \rightarrow \mathbb{N} \vdash F() : {}} \color{red}{Q}$$

$$\hphantom{F : \mathbb{U} \rightarrow {}} \color{green}{Q}$$

Game Semantics

$$F \quad : \quad \mathbb{U} \quad \to \quad \mathbb{N} \quad \vdash \quad F() \quad : \quad \mathbb{N}$$

F	:	$\mathbb{U}$	$\to$	$\mathbb{N}$	$\vdash$	$F()$	:	$\mathbb{N}$
								Q
				Q				
		Q						

Game Semantics

$$
\begin{array}{ccccccccc}
F & : & \mathbb{U} & \rightarrow & \mathbb{N} & \vdash & F() & : & \mathbb{N} \\
 & & & & & & & & \color{red}{Q} \\
 & & & & \color{green}{Q} & & & & \\
 & & \color{red}{Q} & & & & & & \\
 & & \color{green}{()} & & & & & &
\end{array}
$$

Game Semantics

F	$:$	$\mathbb{U}$	$\rightarrow$	$\mathbb{N}$	$\vdash$	$F()$	$:$	$\mathbb{N}$
								Q
				Q				
		Q						
		()						
				4				

Game Semantics

F	$:$	$\mathbb{U}$	$\rightarrow$	$\mathbb{N}$	$\vdash$	$F\,()$	$:$	$\mathbb{N}$
								Q
				Q				
		Q						
		()						
				4				
								4

Game Semantics

F	:	$\mathbb{U}$	$\rightarrow$	$\mathbb{N}$	$\vdash$	$F\,()$	:	$\mathbb{N}$
								Q
				Q				
		Q						
		()						
				4				
								4

- J. M. E. Hyland and C.-H. Luke Ong. On full abstraction for PCF: I, II, and III. *Inf. Comput.*, 163(2):285–408, 2000
- Samson Abramsky, Radha Jagadeesan, and Pasquale Malacaria. Full abstraction for PCF. *Inf. Comput.*, 163(2):409–470, 2000

Concurrent Game Semantics

$$x : \mathbb{U} \quad , \quad y : \mathbb{U} \quad \vdash x \parallel y \quad : \quad \mathbb{U}$$

Q

Concurrent Game Semantics

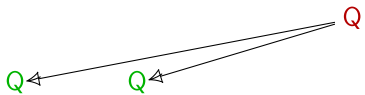
$$x : \mathbb{U} \quad , \quad y : \mathbb{U} \quad \vdash x \parallel y \quad : \quad \mathbb{U}$$

Q

Q Q

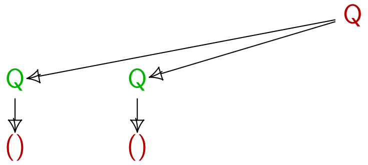
Concurrent Game Semantics

$$x : \mathbb{U} \quad , \quad y : \mathbb{U} \quad \vdash x \parallel y \quad : \quad \mathbb{U}$$



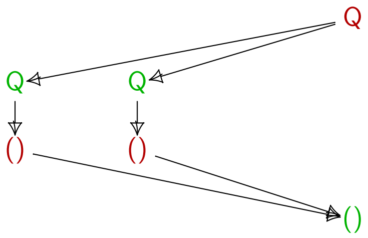
Concurrent Game Semantics

$$x : \mathbb{U} \quad , \quad y : \mathbb{U} \quad \vdash x \parallel y \quad : \quad \mathbb{U}$$



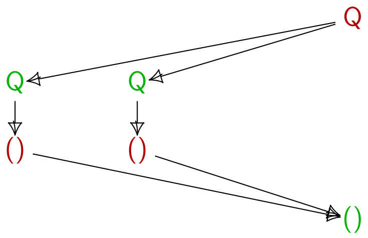
Concurrent Game Semantics

$$x : \mathbb{U} \quad , \quad y : \mathbb{U} \quad \vdash x \parallel y \quad : \quad \mathbb{U}$$



Concurrent Game Semantics

$$x : \mathbb{U} \quad , \quad y : \mathbb{U} \quad \vdash x \parallel y \quad : \quad \mathbb{U}$$



- Silvain Rideau and Glynn Winskel. Concurrent strategies.

  In *Proceedings of the 26th Annual IEEE Symposium on Logic in Computer Science, LICS 2011, June 21-24, 2011, Toronto, Ontario, Canada*, pages 409–418, 2011

- Simon Castellan, Pierre Clairambault, and Glynn Winskel. Thin games with symmetry and concurrent Hyland-Ong games.

  Log. Methods Comput. Sci., 15(1), 2019

Concurrent Game Semantics of Idealized Concurrent Algol

$$F : \mathbb{U} \to \mathbb{U} \vdash \left(\begin{array}{l} \mathsf{newref}\ x\ \mathsf{in} \\ \mathsf{newref}\ y\ \mathsf{in} \\ F\,(x :=!y; \\ \quad F\,(y := 1)); \\ !x \end{array} \right) : \mathbb{N}$$

Concurrent Game Semantics of Idealized Concurrent Algol

$$\left[\!\!\left[F : \mathbb{U} \rightarrow \mathbb{U} \vdash \left(\begin{array}{l} \mathsf{newref}\ x\ \mathsf{in} \\ \mathsf{newref}\ y\ \mathsf{in} \\ F\,(x :=\,!y; \\ \quad F\,(y := 1)); \\ !x \end{array} \right) : \mathbb{N} \right]\!\!\right]$$

Concurrent Game Semantics of Idealized Concurrent Algol

$$\left[\!\left[F : \mathbb{U} \to \mathbb{U} \vdash \left(\begin{array}{l} \mathsf{newref}\ x\ \mathsf{in} \\ \mathsf{newref}\ y\ \mathsf{in} \\ F\,(x := !y; \\ \quad F\,(y := 1)); \\ !x \end{array} \right) : \mathbb{N} \right]\!\right] \quad \ni$$

Concurrent Game Semantics of Idealized Concurrent Algol

$$\left[\!\left[F : \mathbb{U}_0 \to \mathbb{U}_1 \vdash \begin{pmatrix} \mathsf{newref}\ x\ \mathsf{in} \\ \mathsf{newref}\ y\ \mathsf{in} \\ F\,(x := !y; \\ \quad F\,(y := 1)); \\ !x \end{pmatrix} : \mathbb{N}_2 \right]\!\right] \ni$$

Concurrent Game Semantics of Idealized Concurrent Algol

Q_2

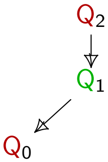
$$\left[\!\!\left[F : \mathbb{U}_0 \to \mathbb{U}_1 \vdash \left(\begin{array}{l} \mathsf{newref}\ x\ \mathsf{in} \\ \mathsf{newref}\ y\ \mathsf{in} \\ F\,(x :=\, !y; \\ \quad F\,(y := 1)); \\ !x \end{array} \right) : \mathbb{N}_2 \right]\!\!\right] \ni$$

Concurrent Game Semantics of Idealized Concurrent Algol

$$\left[\!\!\left[F : \mathbb{U}_0 \to \mathbb{U}_1 \vdash \left(\begin{array}{l} \text{newref } x \text{ in} \\ \text{newref } y \text{ in} \\ F\,(x := !y; \\ \quad F\,(y := 1)); \\ !x \end{array} \right) : \mathbb{N}_2 \right]\!\!\right] \ni$$

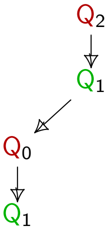
Concurrent Game Semantics of Idealized Concurrent Algol

$$\left[\!\!\left[F : \mathbb{U}_0 \to \mathbb{U}_1 \vdash \begin{pmatrix} \text{newref } x \text{ in} \\ \text{newref } y \text{ in} \\ F\,(x := !y; \\ \quad F\,(y := 1)); \\ !x \end{pmatrix} : \mathbb{N}_2 \right]\!\!\right] \ni$$



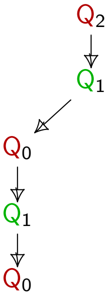
Concurrent Game Semantics of Idealized Concurrent Algol

$$\left[\!\!\left[F : \mathbb{U}_0 \to \mathbb{U}_1 \vdash \left(\begin{array}{l} \mathsf{newref}\ x\ \mathsf{in} \\ \mathsf{newref}\ y\ \mathsf{in} \\ F\,(x := !y; \\ \quad F\,(y := 1)); \\ !x \end{array} \right) : \mathbb{N}_2 \right]\!\!\right] \ni$$



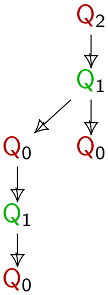
Concurrent Game Semantics of Idealized Concurrent Algol

$$\left[\!\!\left[F : \mathbb{U}_0 \to \mathbb{U}_1 \vdash \left(\begin{array}{l} \text{newref } x \text{ in} \\ \text{newref } y \text{ in} \\ F\,(x := !y; \\ \quad F\,(y := 1)); \\ !x \end{array} \right) : \mathbb{N}_2 \right]\!\!\right] \ni$$



Concurrent Game Semantics of Idealized Concurrent Algol

$$\left[\!\left[F : \mathbb{U}_0 \to \mathbb{U}_1 \vdash \left(\begin{array}{l} \mathsf{newref}\ x\ \mathsf{in} \\ \mathsf{newref}\ y\ \mathsf{in} \\ F\,(x :=\, !y; \\ \quad F\,(y := 1)); \\ !x \end{array} \right) : \mathbb{N}_2 \right]\!\right] \ni$$



Concurrent Game Semantics of Idealized Concurrent Algol

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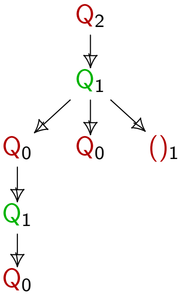
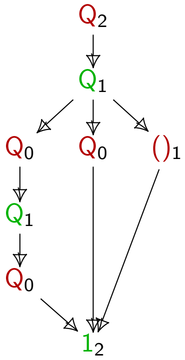


Concurrent Game Semantics of Idealized Concurrent Algol

$$\left[\!\!\left[F : \mathbb{U}_0 \to \mathbb{U}_1 \vdash \begin{pmatrix} \text{newref } x \text{ in} \\ \text{newref } y \text{ in} \\ F\,(x :=!y; \\ \quad F\,(y := 1)); \\ !x \end{pmatrix} : \mathbb{N}_2 \right]\!\!\right] \ni$$



Concurrent Game Semantics

$$\left[\!\!\left[F : \mathbb{U}_0 \to \mathbb{U}_1 \vdash \left(\begin{array}{l} \text{newref } x \text{ in} \\ \text{newref } y \text{ in} \\ F\,(x := !y; \\ \quad F\,(y := 1)); \\ !x \end{array} \right) : \mathbb{N}_2 \right]\!\!\right]$$

Concurrent Game Semantics

$$\left[\!\left[F : \mathbb{U}_0 \to \mathbb{U}_1 \vdash \begin{pmatrix} \mathsf{newref}\ x\ \mathsf{in} \\ \mathsf{newref}\ y\ \mathsf{in} \\ F\,(x :=!y; \\ \quad F\,(y := 1)); \\ !x \end{pmatrix} : \mathbb{N}_2 \right]\!\right]$$

$$= \quad \left[\!\left[\begin{array}{l} F\,(x :=!y; \\ \quad F\,(y := 1)); \\ !x \end{array} \right]\!\right] \circ (\mathrm{id} \otimes \mathsf{cell} \otimes \mathsf{cell})$$

Concurrent Game Semantics

$$\left[\!\!\left[F : \mathbb{U}_0 \to \mathbb{U}_1 \vdash \begin{pmatrix} \mathsf{newref}\ x\ \mathsf{in} \\ \mathsf{newref}\ y\ \mathsf{in} \\ F\,(x :=!y; \\ \quad F\,(y := 1)); \\ !x \end{pmatrix} : \mathbb{N}_2 \right]\!\!\right]$$

$$= \quad \left[\!\!\left[\begin{array}{l} F\,(x :=!y; \\ \quad F\,(y := 1)); \\ !x \end{array} \right]\!\!\right] \circ (\mathrm{id} \otimes \mathsf{cell} \otimes \mathsf{cell})$$

$$= \quad \mathsf{seq} \circ \left(\left[\!\!\left[\begin{array}{l} F\,(x :=!y; \\ \quad F\,(y := 1)) \end{array} \right]\!\!\right] \otimes [\![!x]\!] \right) \circ (\mathrm{id} \otimes \delta \otimes \mathrm{id}) \circ (\mathrm{id} \otimes \mathsf{cell} \otimes \mathsf{cell})$$

Concurrent Game Semantics

$$\left[\!\!\left[F : \mathbb{U}_0 \to \mathbb{U}_1 \vdash \begin{pmatrix} \mathsf{newref}\ x\ \mathsf{in} \\ \mathsf{newref}\ y\ \mathsf{in} \\ F\,(x :=\,!y; \\ \quad F\,(y := 1)); \\ !x \end{pmatrix} : \mathbb{N}_2 \right]\!\!\right]$$

$$= \left[\!\!\left[\begin{array}{l} F\,(x :=\,!y; \\ \quad F\,(y := 1)); \\ !x \end{array} \right]\!\!\right] \circ (\mathrm{id} \otimes \mathsf{cell} \otimes \mathsf{cell})$$

$$= \mathsf{seq} \circ \left(\left[\!\!\left[\begin{array}{l} F\,(x :=\,!y; \\ \quad F\,(y := 1)) \end{array} \right]\!\!\right] \otimes [\![!x]\!] \right) \circ (\mathrm{id} \otimes \delta \otimes \mathrm{id}) \circ (\mathrm{id} \otimes \mathsf{cell} \otimes \mathsf{cell})$$

$$= \mathsf{seq} \circ ((\mathsf{ev} \circ (([\![F]\!] \otimes [\![x :=\,!y;\ F\,(y := 1)]\!]^{!}) \circ (\delta \otimes \mathrm{id} \otimes \mathrm{id}))) \otimes [\![!x]\!]) \circ (\mathrm{id} \otimes \delta \otimes \mathrm{id}) \circ (\mathrm{id} \otimes \mathsf{cell} \otimes \mathsf{cell})$$

Concurrent Game Semantics

$$\left[\!\left[F : \mathbb{U}_0 \to \mathbb{U}_1 \vdash \begin{pmatrix} \mathsf{newref}\ x\ \mathsf{in} \\ \mathsf{newref}\ y\ \mathsf{in} \\ F\,(x := !y; \\ \quad F\,(y := 1)); \\ !x \end{pmatrix} : \mathbb{N}_2 \right]\!\right]$$

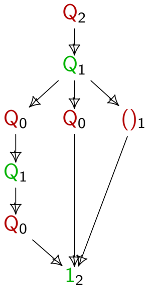
$$= \left[\!\left[\begin{array}{l} F\,(x := !y; \\ \quad F\,(y := 1)); \\ !x \end{array} \right]\!\right] \circ (\mathrm{id} \otimes \mathsf{cell} \otimes \mathsf{cell})$$

$$= \mathsf{seq} \circ \left(\left[\!\left[\begin{array}{l} F\,(x := !y; \\ \quad F\,(y := 1)) \end{array} \right]\!\right] \otimes [\![!x]\!] \right) \circ (\mathrm{id} \otimes \delta \otimes \mathrm{id}) \circ (\mathrm{id} \otimes \mathsf{cell} \otimes \mathsf{cell})$$

$$= \mathsf{seq} \circ ((\mathsf{ev} \circ (([\![F]\!] \otimes [\![x := !y;\ F\,(y := 1)]\!]^{!}) \circ (\delta \otimes \mathrm{id} \otimes \mathrm{id}))) \otimes [\![!x]\!]) \circ (\mathrm{id} \otimes \delta \otimes \mathrm{id}) \circ (\mathrm{id} \otimes \mathsf{cell} \otimes \mathsf{cell})$$

$$= \ldots$$

Concurrent Game Semantics



$\pitchfork$?

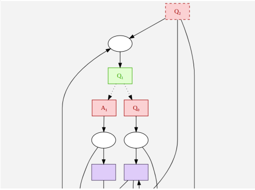
$$\mathsf{seq} \circ \left((\mathsf{ev} \circ ((\llbracket F \rrbracket \otimes \llbracket x := !y;\ F\,(y := 1) \rrbracket^{!}) \circ (\delta \otimes \mathrm{id} \otimes \mathrm{id}))) \otimes \llbracket !x \rrbracket \right) \circ (\mathrm{id} \otimes \delta \otimes \mathrm{id}) \circ (\mathrm{id} \otimes \mathsf{cell} \otimes \mathsf{cell})$$

Contributions

$$\left(\begin{array}{l} \text{newref } x \text{ in} \\ \text{newref } y \text{ in} \\ F\,(x :=!y; \\ \quad F\,(y := 1)); \\ !x \end{array}\right)^{\star}$$

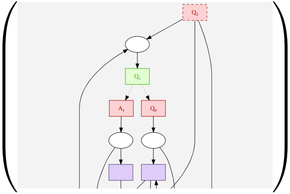
Contributions

$$\left(\begin{array}{l} \text{newref } x \text{ in} \\ \text{newref } y \text{ in} \\ F\,(x := !y; \\ \quad F\,(y := 1)); \\ !x \end{array}\right)^{\star} \quad = \quad \left(\quad \right)$$

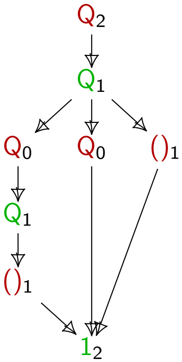


Contributions

$$\left(\begin{array}{l} \text{newref } x \text{ in} \\ \text{newref } y \text{ in} \\ F\,(x := !y; \\ \quad F\,(y := 1)); \\ !x \end{array}\right)^{\star} =$$

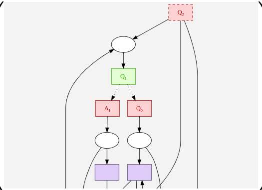


token game $\rightsquigarrow$

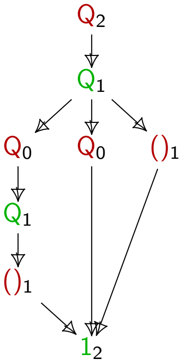


Contributions

$$\left(\begin{array}{l}\text{newref } x \text{ in} \\ \text{newref } y \text{ in} \\ F\,(x := !y; \\ \quad F\,(y := 1)); \\ !x\end{array}\right)^{\star} =$$



token game $\rightsquigarrow$

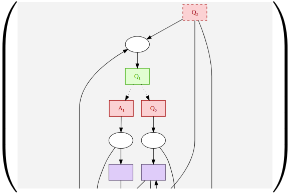


Theorem

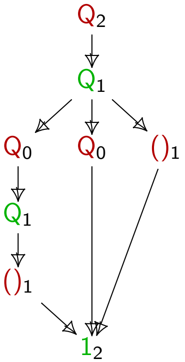
The strategy obtained denotationally, coincides *with that that generated* operationally *by playing the token game played on the intermediate representation.*

Contributions

$$\left(\begin{array}{l} \text{newref } x \text{ in} \\ \text{newref } y \text{ in} \\ F\,(x := !y; \\ \quad F\,(y := 1)); \\ !x \end{array}\right)^{\star} =$$



token game $\rightsquigarrow$



Theorem

The strategy obtained denotationally, coincides *with that that generated* operationally *by playing the token game played on the intermediate representation.*

`ipatopetrinets.github.io`

II. Tools and Methodology

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Geometry of Interaction and **Petri Net Unfoldings**

II. Tools and Methodology

Geometry of Interaction

II. Tools and Methodology

Geometry of Interaction

- Jean-Yves Girard. Geometry of Interaction 1: Interpretation of System F.
  In *Studies in Logic and the Foundations of Mathematics*, volume 127, pages 221–260.
  Elsevier, 1989

Geometry of Interaction : Token Machines

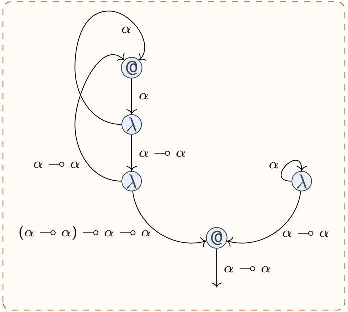
$$(\lambda f^{\alpha \multimap \alpha}.\, \lambda x^{\alpha}.\, f\, x)\, (\lambda y^{\alpha}.\, y)$$

Geometry of Interaction : Token Machines

$$((\lambda f^{\alpha \multimap \alpha}.\, \lambda x^{\alpha}.\, f\, x)\,(\lambda y^{\alpha}.\, y))^{\star} \qquad =$$

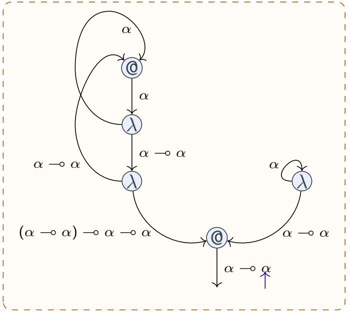
Geometry of Interaction : Token Machines

$$((\lambda f^{\alpha \multimap \alpha}.\, \lambda x^{\alpha}.\, f\, x)\, (\lambda y^{\alpha}.\, y))^{\star} \quad =$$

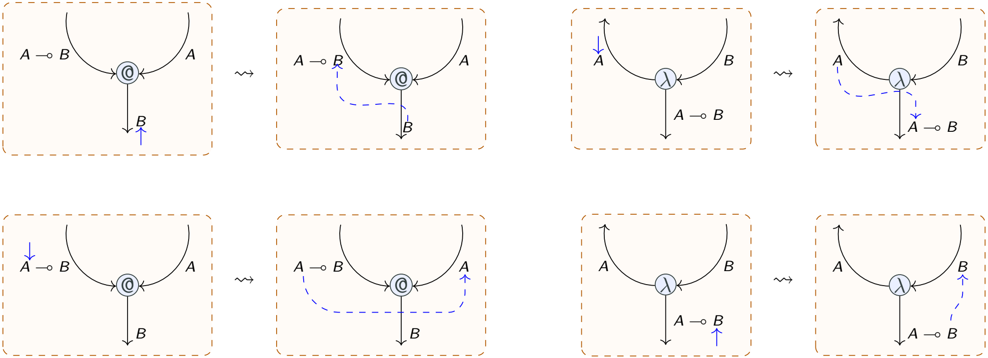


Geometry of Interaction : Token Machines

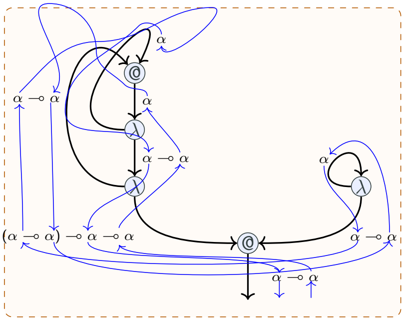
$$((\lambda f^{\alpha \multimap \alpha}.\, \lambda x^{\alpha}.\, f\, x)\, (\lambda y^{\alpha}.\, y))^{\star} \qquad =$$



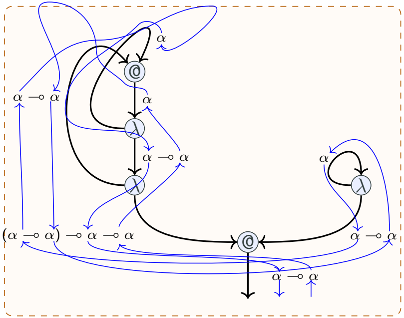
Geometry of Interaction : Token Machines



Geometry of Interaction : Token Machines

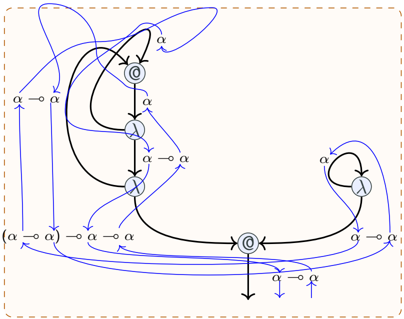


Geometry of Interaction : Token Machines

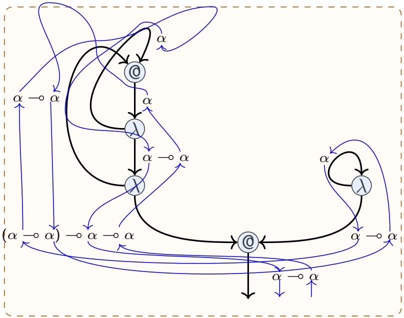


- Ian Mackie. The geometry of interaction machine.
  In Ron K. Cytron and Peter Lee, editors, *POPL 1995*, pages 198–208. ACM Press, 1995
- Vincent Danos, Hugo Herbelin, and Laurent Regnier. Game semantics & abstract machines.
  In *Proceedings, 11th Annual IEEE Symposium on Logic in Computer Science, New Brunswick, New Jersey, USA, July 27-30, 1996*, pages 394–405. IEEE Computer Society, 1996

Token Machines and Game Semantics



Token Machines and Game Semantics

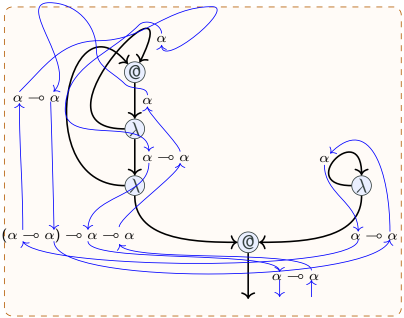


$\alpha \quad \multimap \quad \alpha$

Q

Q

Token Machines and Game Semantics

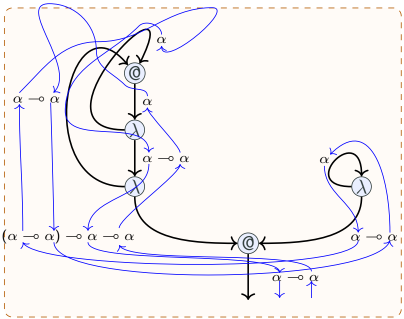


α	$\multimap$	α
		Q
Q		

Theorem (Baillot, 1999)

For Intuitionistic Multiplicative Exponential Linear Logic, the GoI token machine "generates" the AJM game semantics.

Token Machines and Game Semantics



$\alpha \quad \multimap \quad \alpha$

Q

Q

Theorem (Baillot, 1999)

For Intuitionistic Multiplicative Exponential Linear Logic, the GoI token machine "generates" the AJM game semantics.

- Patrick Baillot. *Approches dynamiques en sémantique de la logique linéaire: jeux et géométrie de l'interaction.*
  PhD thesis, Aix-Marseille 2, 1999

Idealized Concurrent Algol (ICA)

A **call-by-name**, **higher-order concurrent** language with **shared memory**:

M, N	$::=$	$\lambda x.\, M \mid M\, N \mid x \mid \mathsf{Y}$	**λ-calculus + recursion**
		$\mid \mathtt{tt} \mid \mathtt{ff} \mid \mathsf{if}\ M\ N_1\ N_2$	**booleans**
		$\mid n \mid \mathsf{succ}\ M \mid \mathsf{pred}\ M \mid \mathsf{iszero}\ M$	**natural numbers**
		$\mid \mathsf{skip} \mid M;\ N \mid M \parallel N$	**commands**
		$\mid \mathsf{newref}\ x\ \mathsf{in}\ M \mid\ !M \mid M := N$	**references**
		$\mid \mathsf{newsem}\ x\ \mathsf{in}\ M \mid \mathsf{grab}\ M \mid \mathsf{release}\ M$	**semaphores**
		$\mid \mathsf{let}\ x = M\ \mathsf{in}\ N$	**let binding**

II. Tools and Methodology

Geometry of Interaction and **Petri Net Unfoldings**

II. Tools and Methodology

Petri Net Unfoldings

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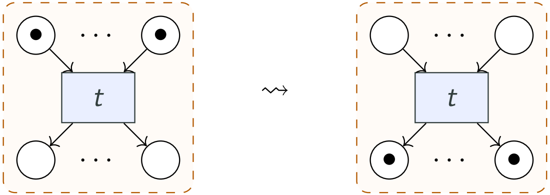
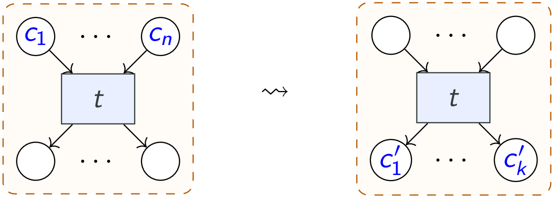
Petri Net Unfoldings

- Mogens Nielsen, Gordon D. Plotkin, and Glynn Winskel. Petri nets, event structures and domains.
  In *Semantics of Concurrent Computation*, volume 70 of *Lecture Notes in Computer Science*, pages 266–284. Springer, 1979

Petri Nets and their Unfolding

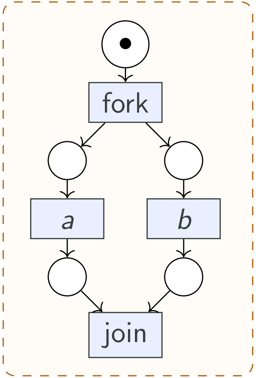


Petri Nets and their Unfolding

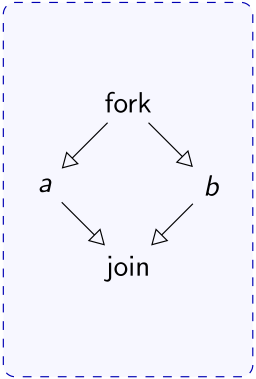


$$\delta\langle t\rangle(c_1, \ldots, c_n) = (c'_1, \ldots, c'_k)$$

Petri Nets and their Unfolding

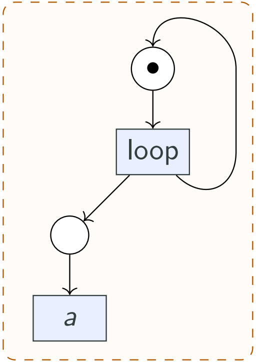


Petri net

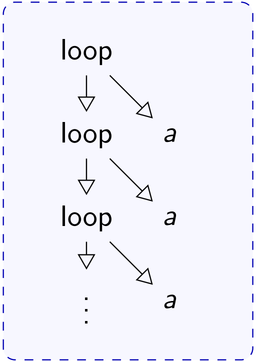


Unfolding

Petri Nets and their Unfolding



Petri net

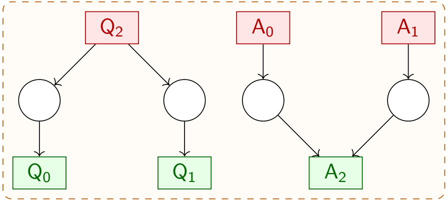


Unfolding

III. Computing Petri Nets Compositionally

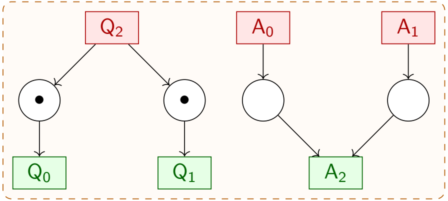
Interactive Petri Nets

$$x : \mathbb{U}_0, y : \mathbb{U}_1 \vdash x \parallel y : \mathbb{U}_2$$



Interactive Petri Nets

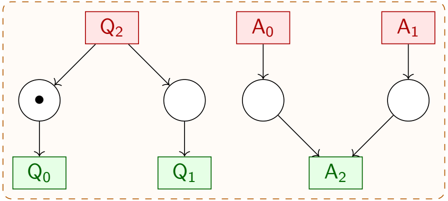
$$x : \mathbb{U}_0, y : \mathbb{U}_1 \vdash x \parallel y : \mathbb{U}_2$$

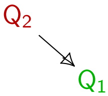


Q_2

Interactive Petri Nets

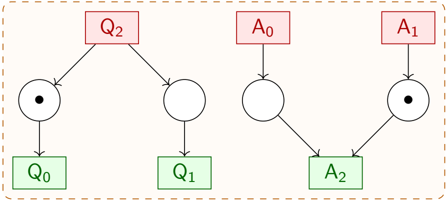
$$x : \mathbb{U}_0, y : \mathbb{U}_1 \vdash x \parallel y : \mathbb{U}_2$$

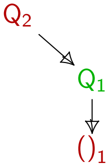




Interactive Petri Nets

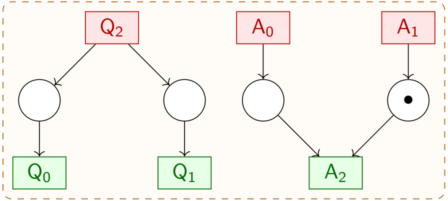
$$x : \mathbb{U}_0, y : \mathbb{U}_1 \vdash x \parallel y : \mathbb{U}_2$$

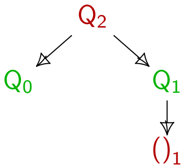




Interactive Petri Nets

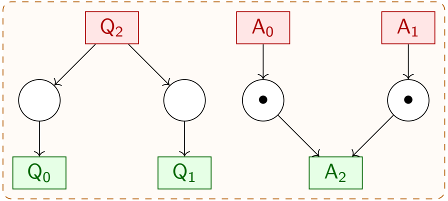
$$x : \mathbb{U}_0, y : \mathbb{U}_1 \vdash x \parallel y : \mathbb{U}_2$$

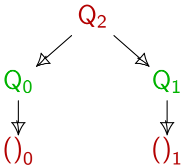




Interactive Petri Nets

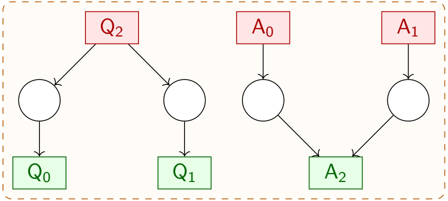
$$x : \mathbb{U}_0, y : \mathbb{U}_1 \vdash x \parallel y : \mathbb{U}_2$$

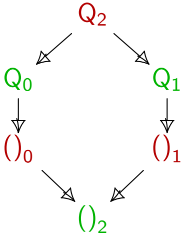




Interactive Petri Nets

$$x : \mathbb{U}_0, y : \mathbb{U}_1 \vdash x \parallel y : \mathbb{U}_2$$



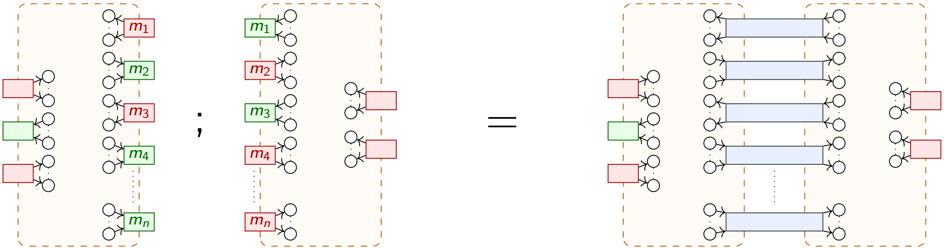


Petri Strategies

Definition

A **Petri strategy** on game A is an interactive Petri net which "obeys the rules of A", *i.e.* which **unfolds** to a **concurrent strategy** on A.

Composition of Petri Strategies



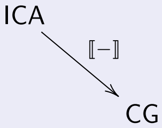
Petri Strategies

Theorem

ICA

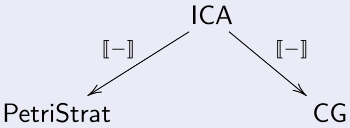
Petri Strategies

Theorem



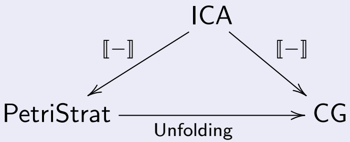
Petri Strategies

Theorem

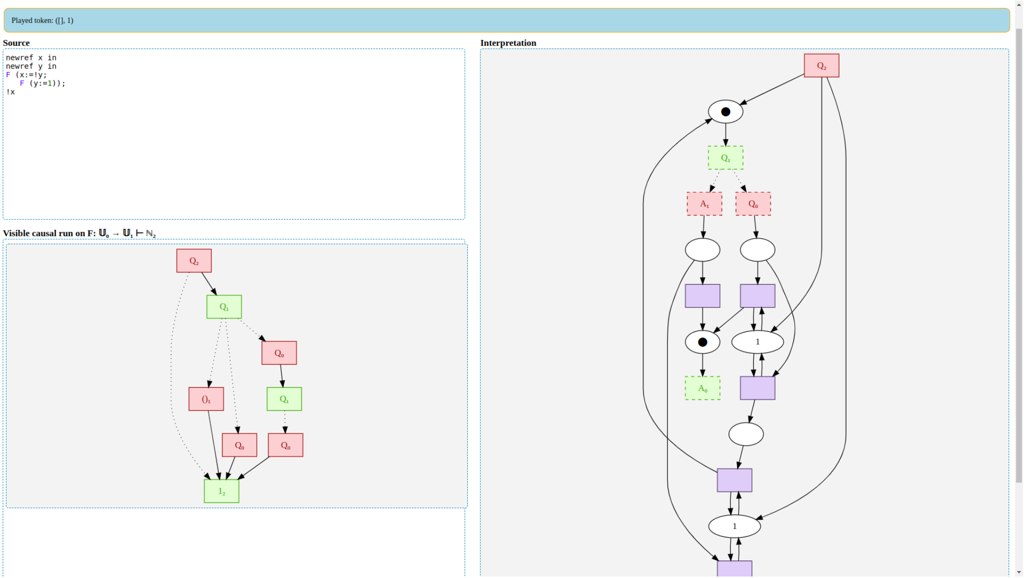


Petri Strategies

Theorem



Implementation: ipatopetrinets.github.io



V. Conclusions

Conclusions and Perspectives

Contributions:

- A GoI multi-token machine for ICA,
- A new methodology to develop/prove correct token machines,
- A powerful bridge between operational and denotational semantics,
- An implementation:

  `ipatopetrinets.github.io`

Future work:

- More realistic languages (call-by-value),
- Applications to verification,
- ??