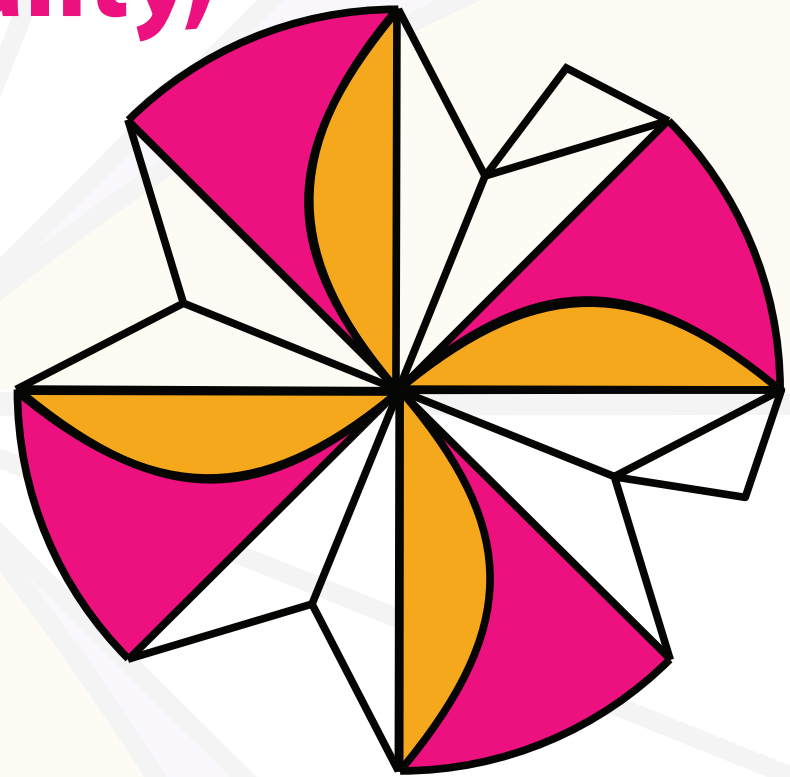
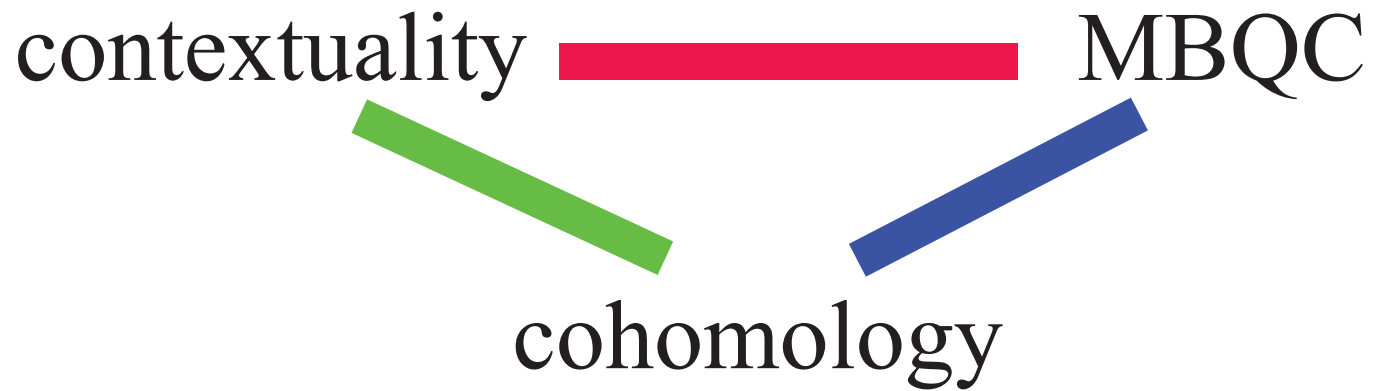


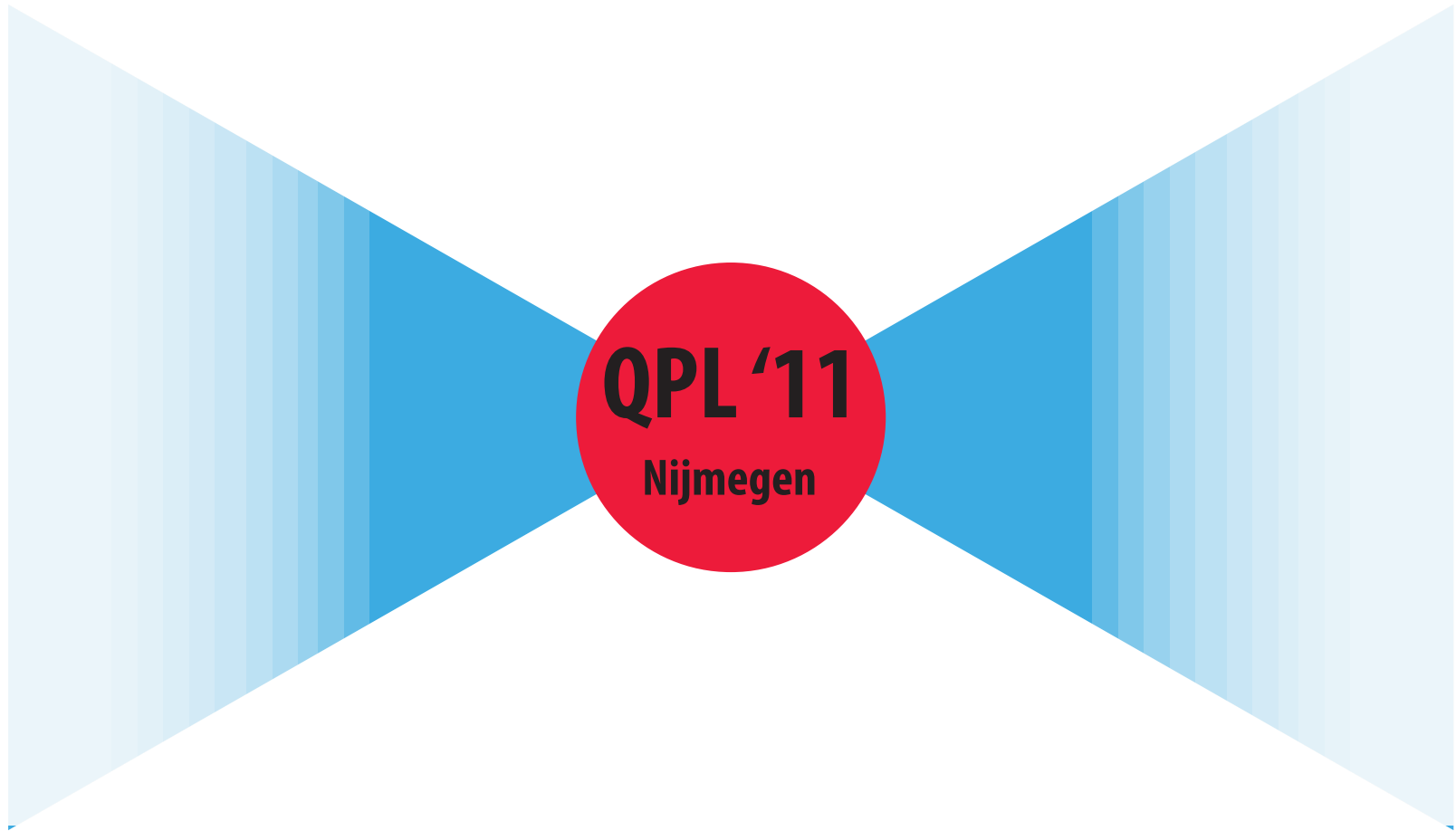
Interwoven paths: my journey through contextuality, cohomology and paradox



Robert Raussendorf, Leibniz Universität Hannover
University College London, September 2023

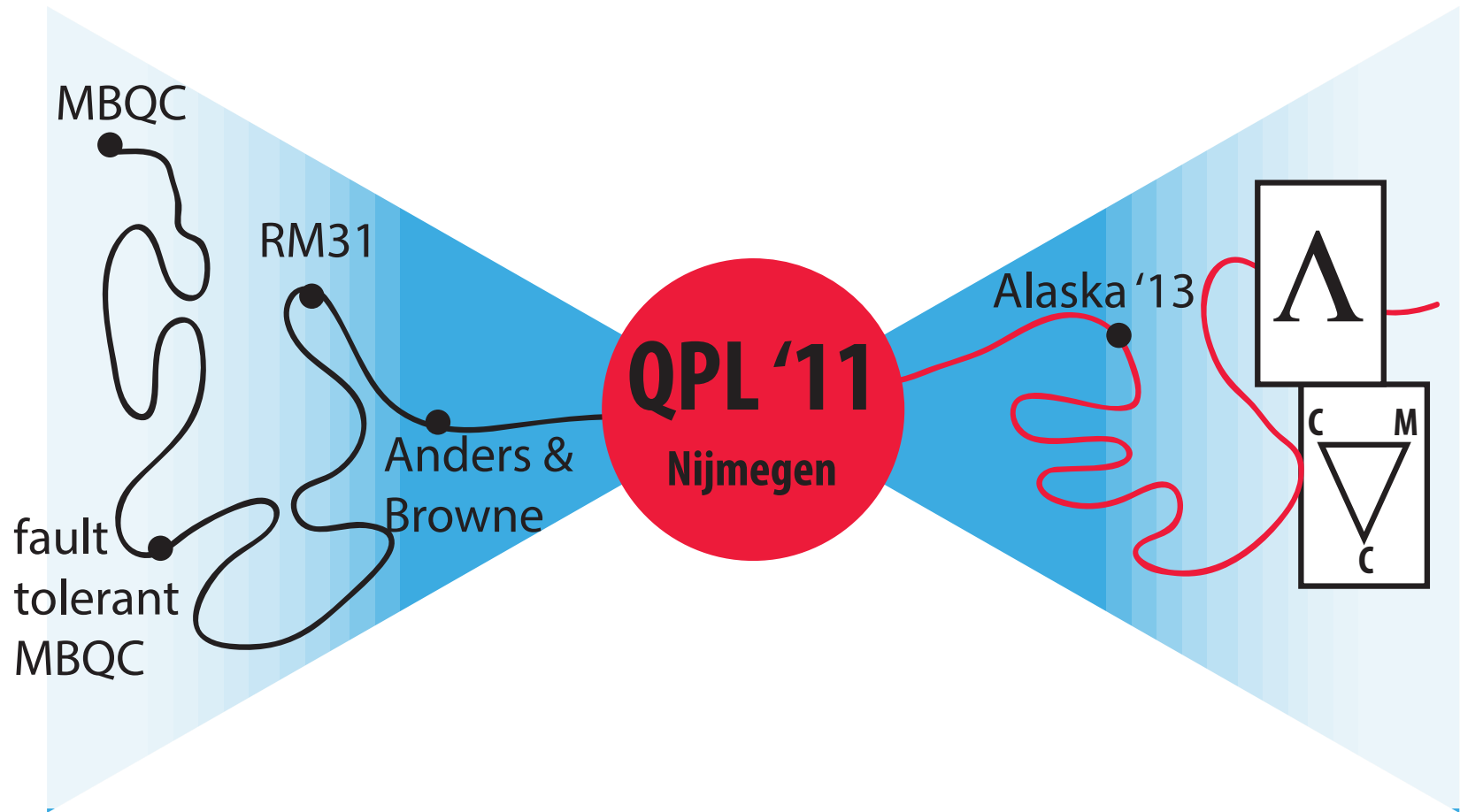


The contextuality – MBQC – cohomology triangle



6 Travel log

As I learned over the years, the 8th Conference on Quantum Physics and Logic, held in Nijmegen, the Netherlands in November 2011, is remembered fondly by many participants; for all sorts of reasons. Here I'd like to describe my journey towards this conference, how I spiralled out of it, and my thoughts for the future.



6 Travel log

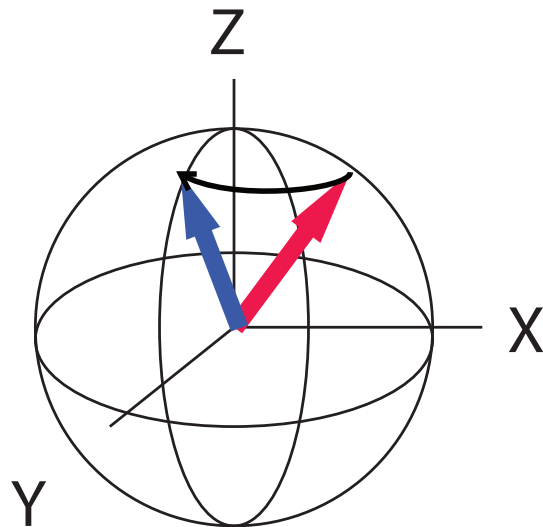
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Measurement-based
quantum computation

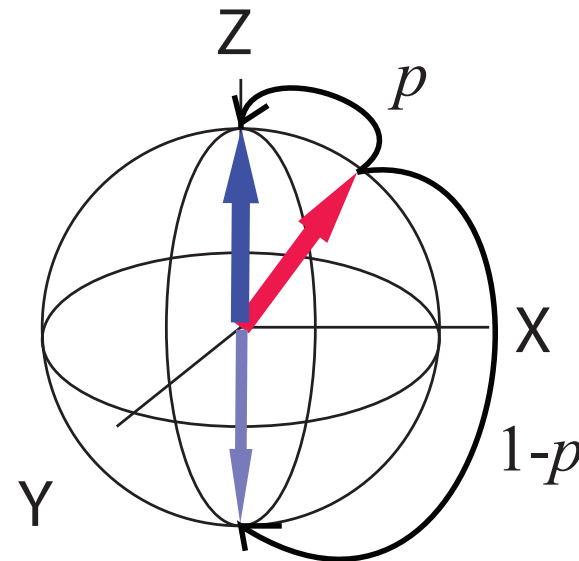
Measurement-based quantum computation

Unitary transformation



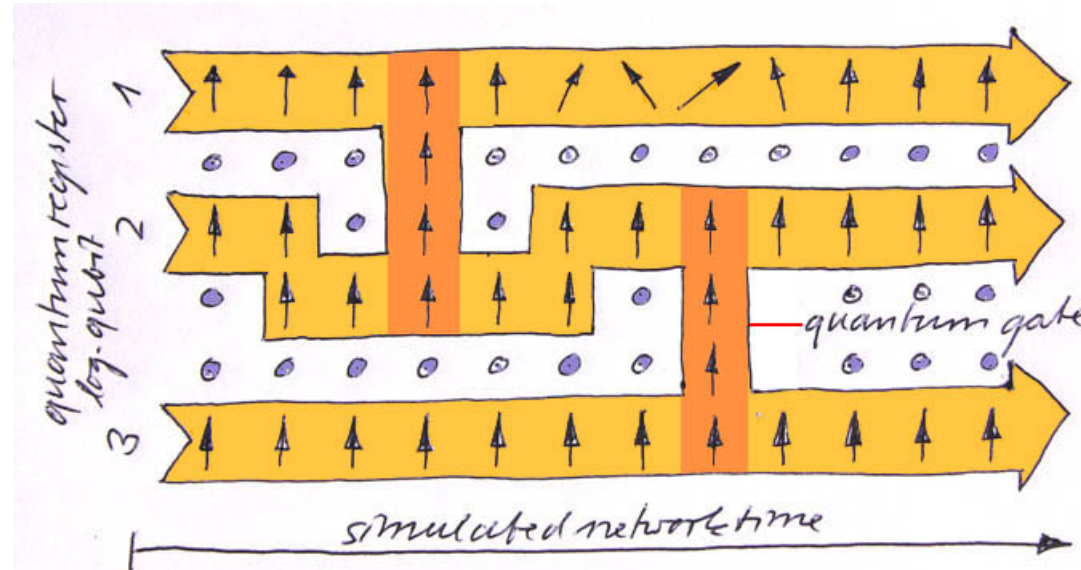
deterministic,
reversible

Projective measurement



probabilistic,
irreversible

Measurement-based quantum computation

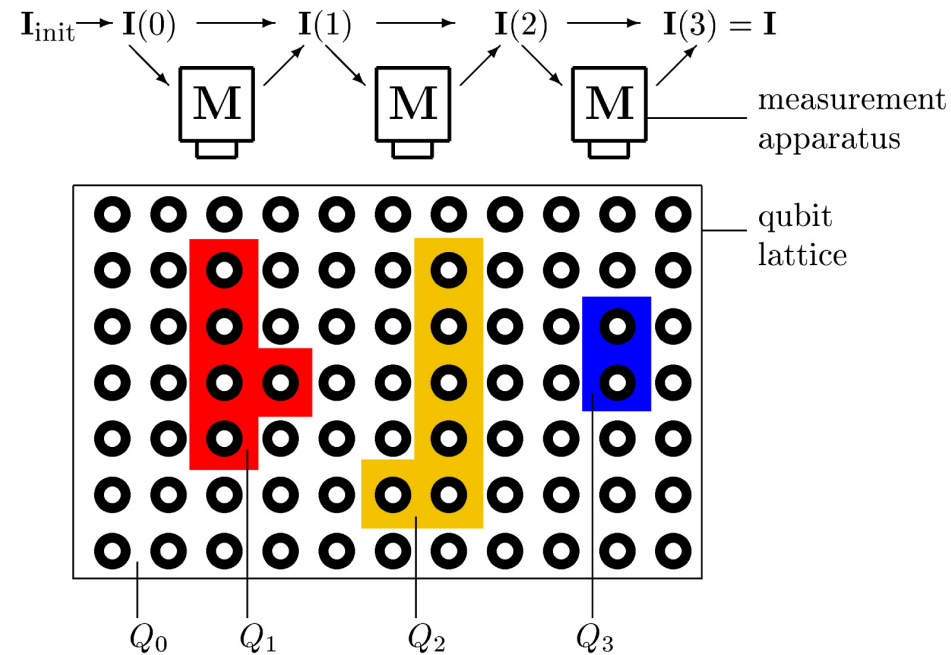


measurement of Z (\odot), X (\uparrow), $\cos \alpha X + \sin \alpha Y$ (\nearrow)

- Information written onto the resource state, processed and read out by one-qubit measurements only.
- Universal computational resources exist: cluster state, AKLT state.

R. Raussendorf, H.-J. Briegel, Physical Review Letters 86, 5188 (2001).

Measurement-based quantum computation



- The outcome bits of the computations are *correlations* among measurement outcomes.

Correlations ferreted out by *linear* classical side processing.

R. Raussendorf and H.J. Briegel, *Computational model underlying the one-way quantum computer*, Quant. Inf. Comp. 6, 443 (2002).



Fault tolerant
measurement-based
quantum computation



New Years Card 2004



Progress up to 2023

Fault-tolerant MBQC

- I expected: Fault-tolerance in MBQC could only be resolved if we understood the non-Pauli correlations in MBQC.

Solving fault-tolerance for MBQC would combine the interesting with the useful — a goldilocks problem.

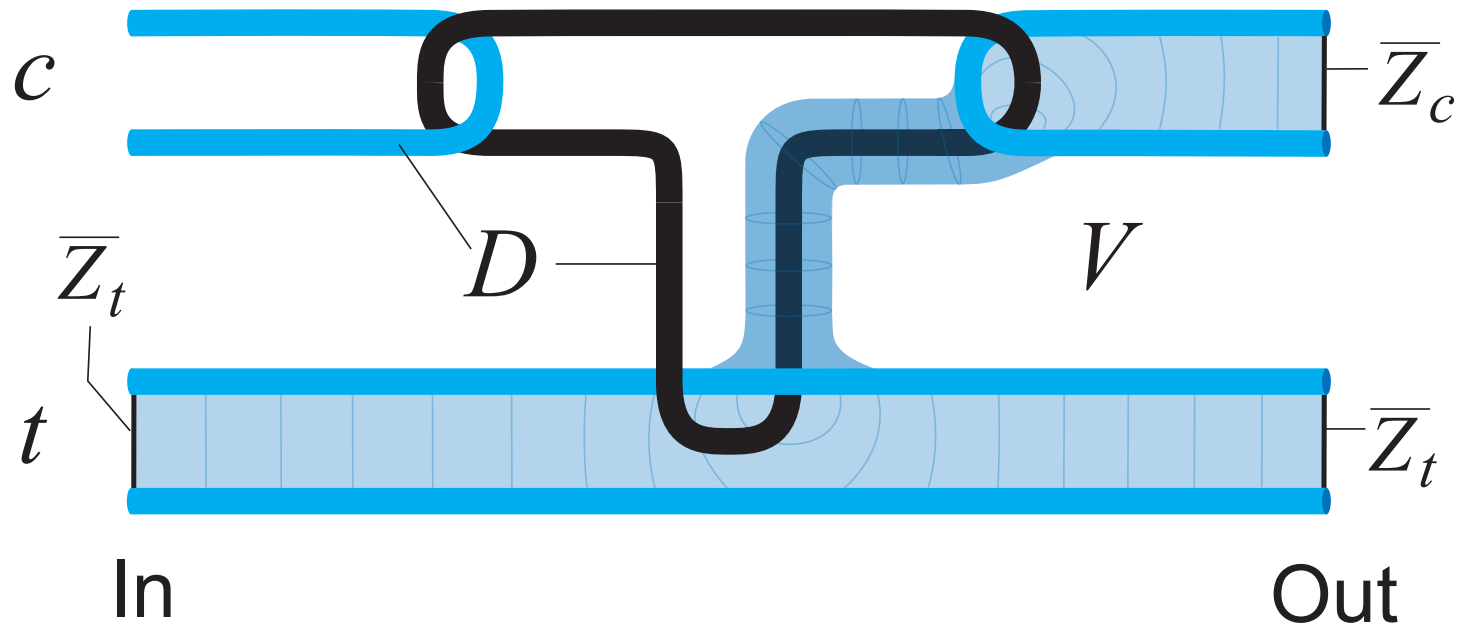
- I anticipated: first construction would be cumbersome, and fail.

- 2005: We solved it!

-



Fault-tolerant MBQC



Topologically protected CNOT gate in 3D cluster states

R. Raussendorf, J. Harrington, K. Goyal, Ann. Phys. (N.Y.) 321, 2242 (2006).

R. Raussendorf and J. Harrington, Phys. Rev. Lett. 98, 190504 (2007).

Fault-tolerant MBQC

- I expected: Fault-tolerance in MBQC could only be resolved if we understood the non-Pauli correlations in MBQC.

Solving fault-tolerance for MBQC would combine the interesting with the useful — a goldilocks problem.

- I anticipated: first construction would be cumbersome, and fail.
- 2005: We solved it!
- *The non-Pauli correlations did not need to be understood to solve fault-tolerance for MBQC.*





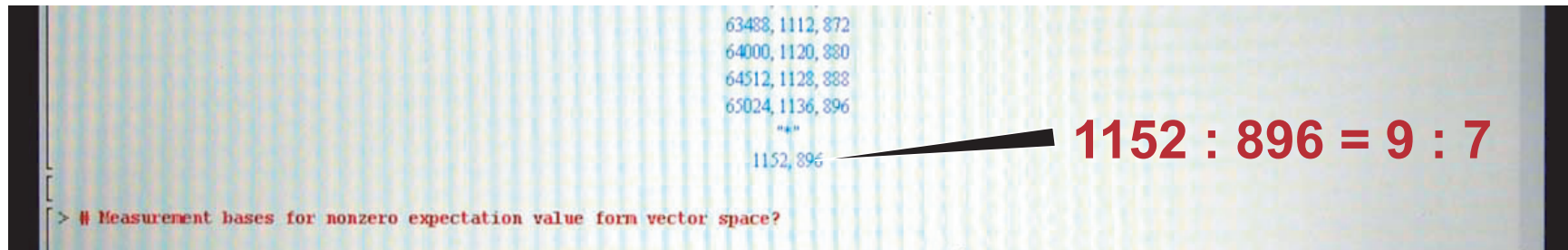
Sergey Bravyi and I shared an office at IQI
Sergey \rightarrow magic state distillation \rightarrow Reed-Muller codes



RM31 *

*: for Reed-Muller

Why not use RM code states for MBQC?



A screenshot of a terminal window displaying Reed-Muller code parameters. The text includes a list of coordinates: 63488, 1112, 872; 64000, 1120, 880; 64512, 1128, 888; 65024, 1136, 896; and 1152, 896. A red arrow points from the last coordinate to the red text $1152 : 896 = 9 : 7$. Below the list is a red prompt: `> # Measurement bases for nonzero expectation value form vector space?`

Reed-Muller code states provide MBQC resource states for

- Deterministically computing a non-linear Boolean function,
- While obeying the linear classical side processing relations of MBQC, and
- Being non-Clifford.

All three criteria satisfied for 31 qubits.

(These are toy computations)



Contextuality in MBQC:
Anders & Browne

N. David Mermin

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853-2501

Although skeptical of the prohibitive power of no-hidden-variables theorems, John Bell was himself responsible for the two most important ones. I describe some recent versions of the lesser known of the two (familiar to experts as the “Kochen-Specker theorem”) which have transparently simple proofs. One of the new versions can be converted without additional analysis into a powerful form of the very much better known “Bell’s Theorem,” thereby clarifying the conceptual link between these two results of Bell.

CONTENTS

I. The Dream of Hidden Variables 803
 II. Plausible Constraints on a Hidden-Variables Theory 804
 III. Von Neumann’s Silly Assumption 805
 IV. The Bell-Kochen-Specker Theorem 806
 V. A Simpler Bell-KS Theorem in Four Dimensions 809
 VI. A Simple and More Versatile Bell-KS Theorem in Four Dimensions 809
 VII. Is the Bell-KS Theorem Silly? 810
 VIII. Locality Replaces Noncontextuality: Bell’s Theorem 811
 IX. A Little About Bohm Theory 812
 X. The Last Word 813
 Acknowledgments 813
 References 813

Like all authors of noncommissioned reviews, I am sure that she can restate the position with such clarity and simplicity that all previous discussions will be eclipsed.
 J. S. Bell, 1966

I. THE DREAM OF HIDDEN VARIABLES

It is a fundamental quantum doctrine that a measurement does not, in general, reveal a preexisting value of the measured property. On the contrary, the outcome of a measurement is brought into being by the act of measurement itself, a joint manifestation of the state of the probed system and the probing apparatus. Precisely how the particular result of an individual measurement is brought into being—Heisenberg’s “transition from the possible to the actual”—is inherently unknowable. Only the statistical distribution of many such encounters is a proper matter for scientific inquiry.

We have been told this so often that the eyes glaze over at the words, and half of you have probably stopped reading already. But is it really true? Or, more conservatively, is it really necessary? Does quantum mechanics, that powerful, practical, phenomenally accurate computational tool of physicist, chemist, biologist, and engineer, really demand this weak link between our knowledge and the objects of that knowledge? Setting aside the metaphysics that emerged from urgent debates and long walks in Copenhagen parks, can one point to anything in the modern quantum theory that forces on us such an act of intellectual renunciation? Or is it merely reverence for the Patriarchs that leads us to deny that a measurement reveals a value that was already there, prior to the measurement?

Well, you might say, it’s easy enough to deduce from quantum mechanics that in general the measurement ap-

paratus disturbs the system on which it acts. True so what? One can easily imagine a measurement m up any number of things, while still revealing the v a preexisting property. Ah, you might add, but certainly principle prohibits the existence of joint for certain important groups of physical quantities. I might the Patriarchs, but as deduced

ensemble. But surely is built into the quantum theory. Entirely built into the quantum theory. The question is not whether the measurement disturbs the system, but whether the measurement reveals a preexisting value.

What, in fact, can you say? The celebrated polymath who first refuted the “Most Theoretical Physicists” position was a quantum theorist. He argued that the indeterministic indeterminability of the outcome of a measurement of a subatomic property simply tells us that we cannot know the value of the property at one instant and the value of the property at another instant. He does not tell us what the value is. (Physicists) do not know what is not measured.

Are we, then, to consider the possibility of a world, in which the properties of individual systems do not have simultaneous values? Such deeper levels of description are excluded by the act of measurement. Hidden-variables programs. A frequent claim is that a successful hidden-variables theory is to quantum mechanics as classical statistical mechanics (see, for example, Einstein, in Schilpp, 1949, p. 672): quantum mechanics would survive intact, but would be understood in terms of a deeper and more detailed picture of the world. Efforts, on the other hand, to put our notori-

Computational Power of Correlations

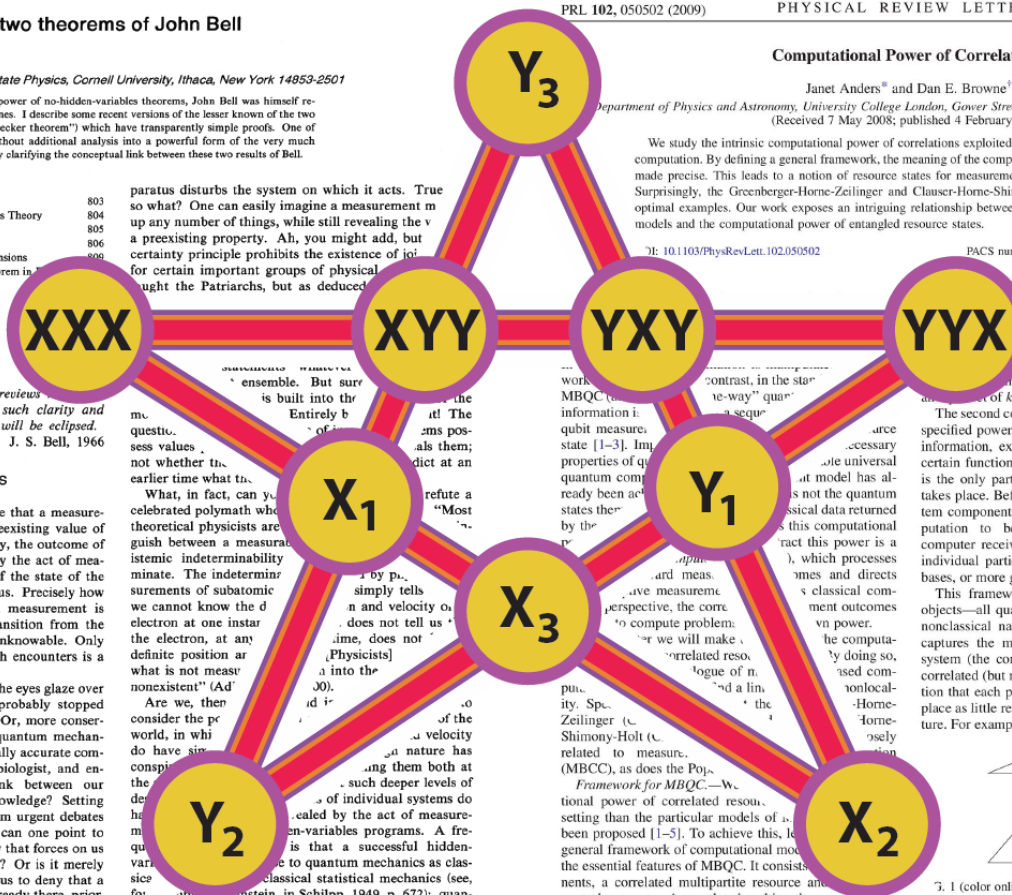
Janet Anders¹ and Dan E. Browne¹

¹Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom (Received 7 May 2008; published 4 February 2009)

We study the intrinsic computational power of correlations exploited in measurement-based quantum computation. By defining a general framework, the meaning of the computational power of correlations is made precise. This leads to a notion of resource states for measurement-based classical computation. Surprisingly, the Greenberger-Horne-Zeilinger and Clauser-Horne-Shimony-Holt problems emerge as optimal examples. Our work exposes an intriguing relationship between the violation of local realistic models and the computational power of entangled resource states.

DOI: 10.1103/PhysRevLett.102.050502

PACS numbers: 03.67.Lx, 03.65.Ud, 89.70.Eg



...ats are solely due to their joint communication between parties is computation. There shall be just a of data with each party. This restriction and we discuss its necessity and [9]. The party will receive an input from of k choices and will return one of l outcomes.

The second component is a classical control computer of specified power. The control computer can store classical information, exchange it with the parties, and compute certain functions. Notably, the classical control computer is the only part of the model where active computation takes place. Before the computation commences, the system components are preprogrammed to specify the computation to be performed. Specifically, the control computer receives the functions it will evaluate and the individual parties receive a specific set of measurement bases, or more generally a choice of k settings.

This framework consists only of explicitly classical objects—all quantum features are hidden in the possibly nonclassical nature of the correlations. The framework captures the most general model of a single classical system (the control computer) interacting with multiple correlated (but nonsignalling) parties, with the key restriction that each party is addressed only once. However, we place as little restriction as possible on their internal structure. For example, the parties making up the system could

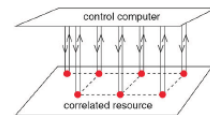
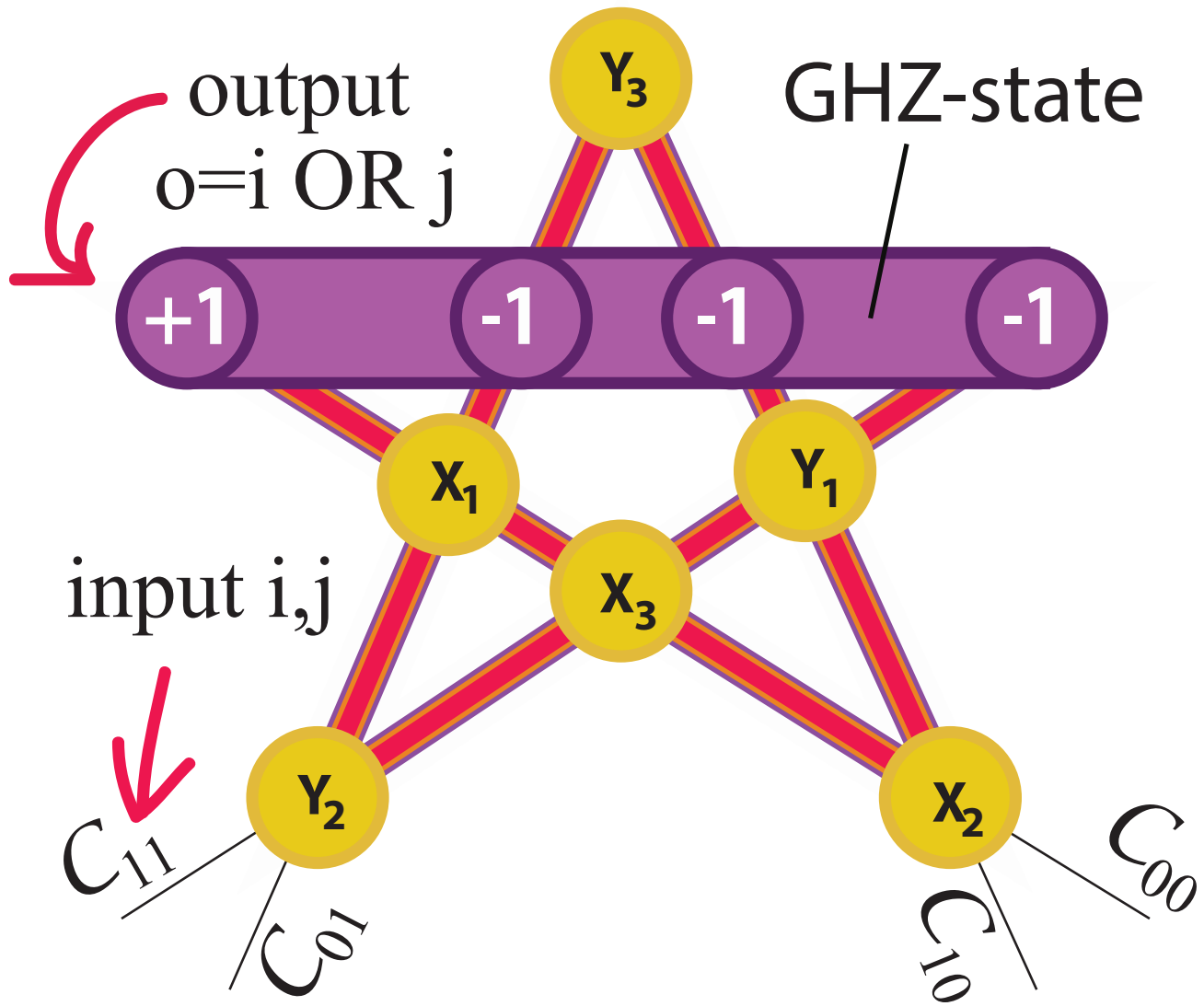


FIG. 1 (color online). The control computer provides one of k choices as the classical input (downward arrows) to each of the correlated parties (circles in the resource) and receives one of l choices as the output.

- Mermin’s star, a contextuality proof on 3 qubits, can be repurposed as an MBQC!



QPL 2011



**Contextuality and Cohomology:
Abramsky, Barbosa, Mansfield**

The Cohomology of Non-Locality and Contextuality

Samson Abramsky

Shane Mansfield

Rui Soares Barbosa

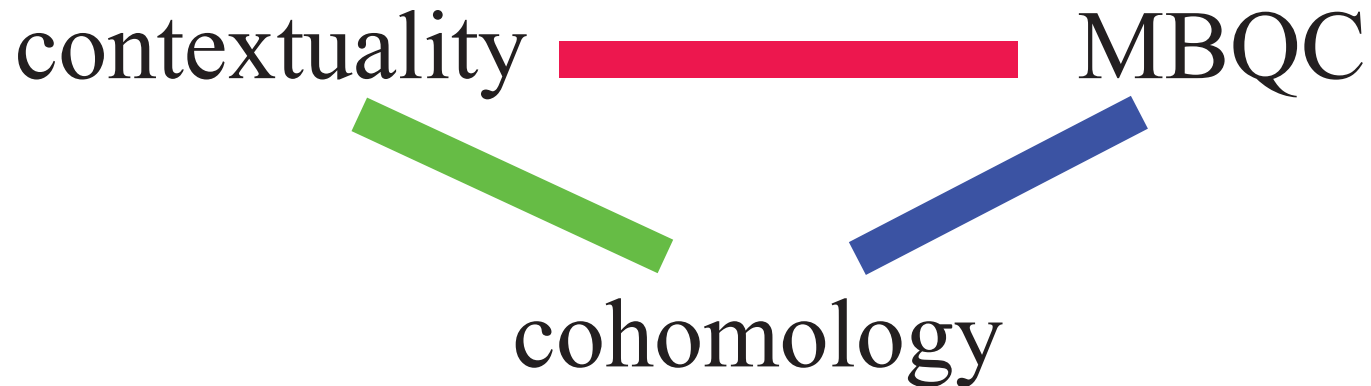
Department of Computer Science
University of Oxford

{samson.abramsky, shane.mansfield, rui.soaresbarbosa}@cs.ox.ac.uk

In a previous paper with Adam Brandenburger, we used sheaf theory to analyze the structure of non-locality and contextuality. Moreover, in this formulation, we showed that the phenomena of non-locality and contextuality are precisely in terms of obstructions to the existence of global sections.



Seemingly in close reach after QPL '11



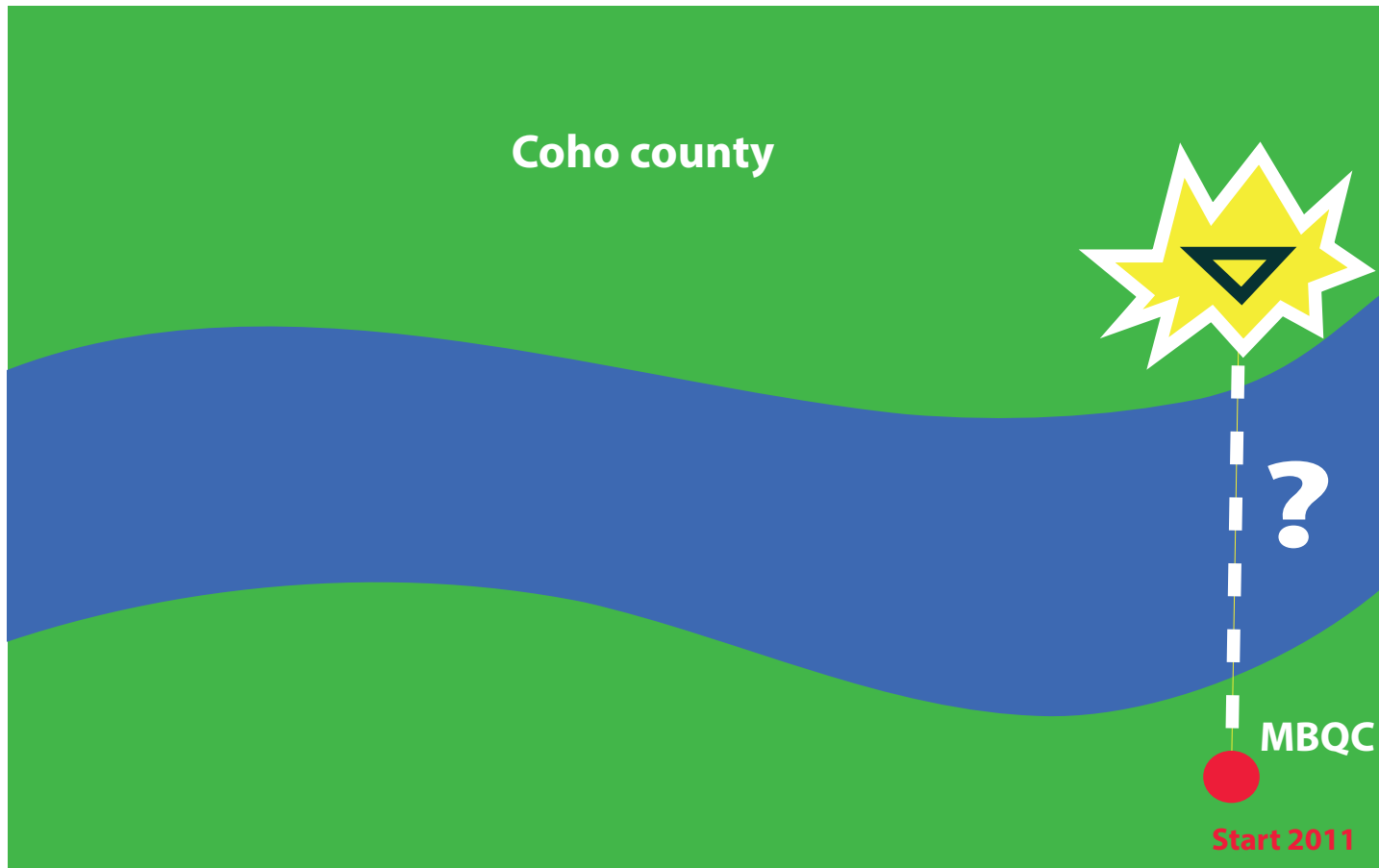
The contextuality – MBQC – cohomology triangle

Partially established (temporally flat MBQCs only)

Inspirations:

Anders and Browne, *Computational Power of Correlations*, PRL 102 (2009),
Abramsky, Barbosa, Mansfield, *Cohomology of contextuality*, arXiv:1111.3620.

But ...



Unlocking the triangle proved to be harder than thought.

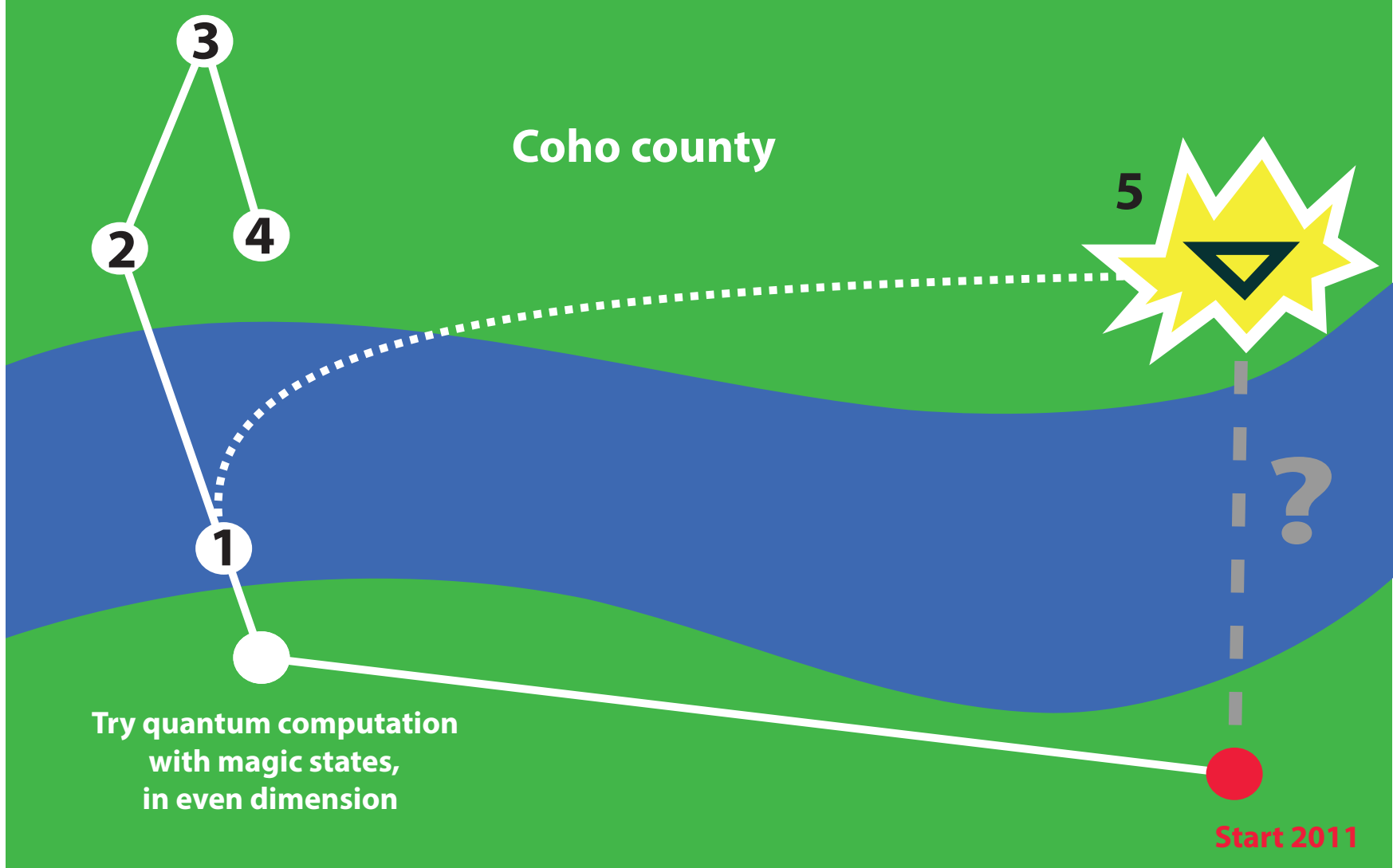


Alaska,
Summer 2013





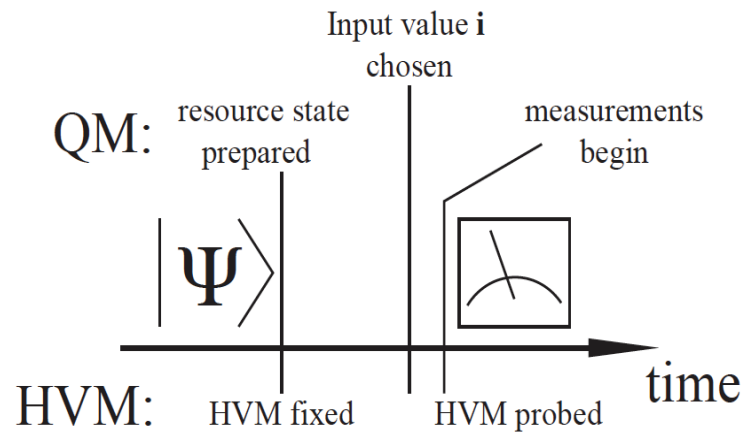
Joseph Emerson and Stephen Bartlett, July 2013



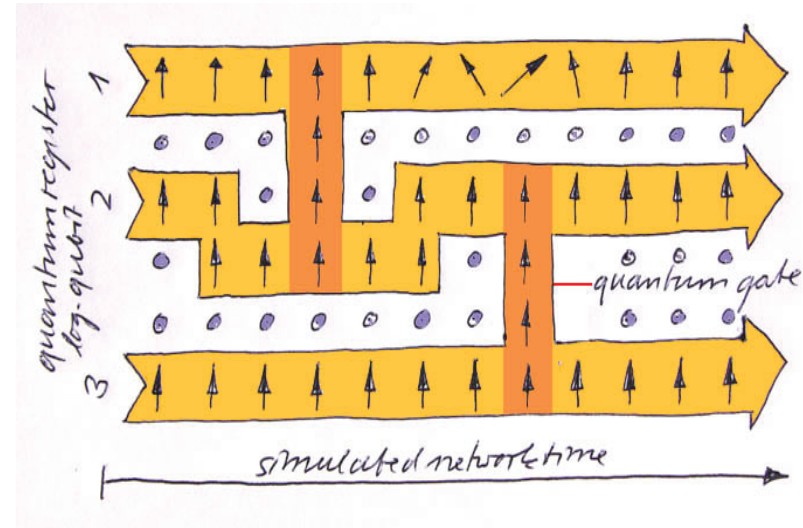


Interaction Picture

(i) Sorting out MBQC \longleftrightarrow contextuality



??



Theorem 1.* An MBQC evaluating a nonlinear Boolean function $o : (\mathbb{Z}_2)^m \longrightarrow \mathbb{Z}_2$ deterministically is contextual.

*: R. Raussendorf, Phys. Rev. A, 022322 (2013).

(i) Sorting out MBQC \longleftrightarrow contextuality

Theorem 2.* Be \mathcal{M} an MBQC evaluating a nonlinear Boolean function $o : (\mathbb{Z}_2)^m \rightarrow \mathbb{Z}_2$ with average success probability p_S . Then, \mathcal{M} is contextual if $p_S > 1 - 1/2^m$, and, for bent functions, if $p_S > 1/2 + 1/2^{m/2+1}$.

Theorem 3.** Be \mathcal{M} an MBQC evaluating a nonlinear Boolean function $o : (\mathbb{Z}_2)^m \rightarrow \mathbb{Z}_2^l$ with average success probability p_S . Then,

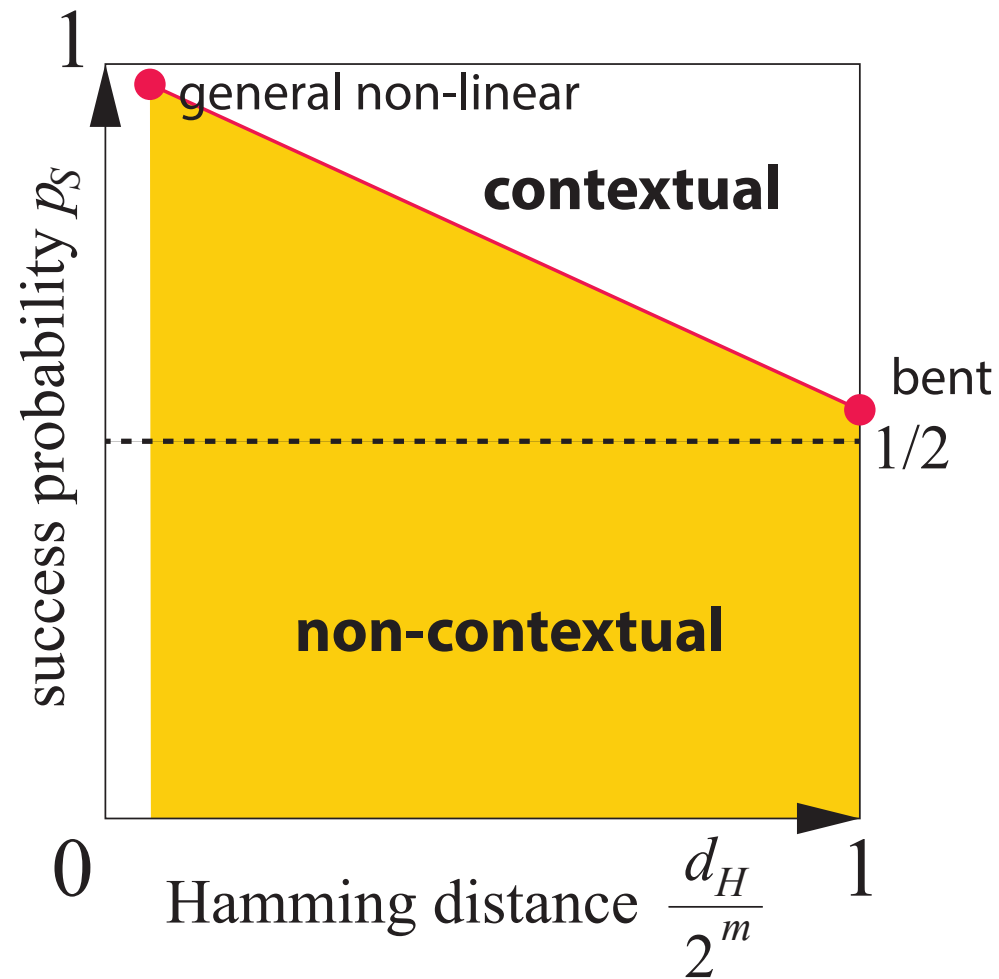
$$p_S \leq 1 - \text{NCF} \frac{d_H(o)}{2^m}.$$

Therein, $d_H(o)$ is the Hamming distance from the closest linear function.

*: R. Raussendorf, Phys. Rev. A, 022322 (2013).

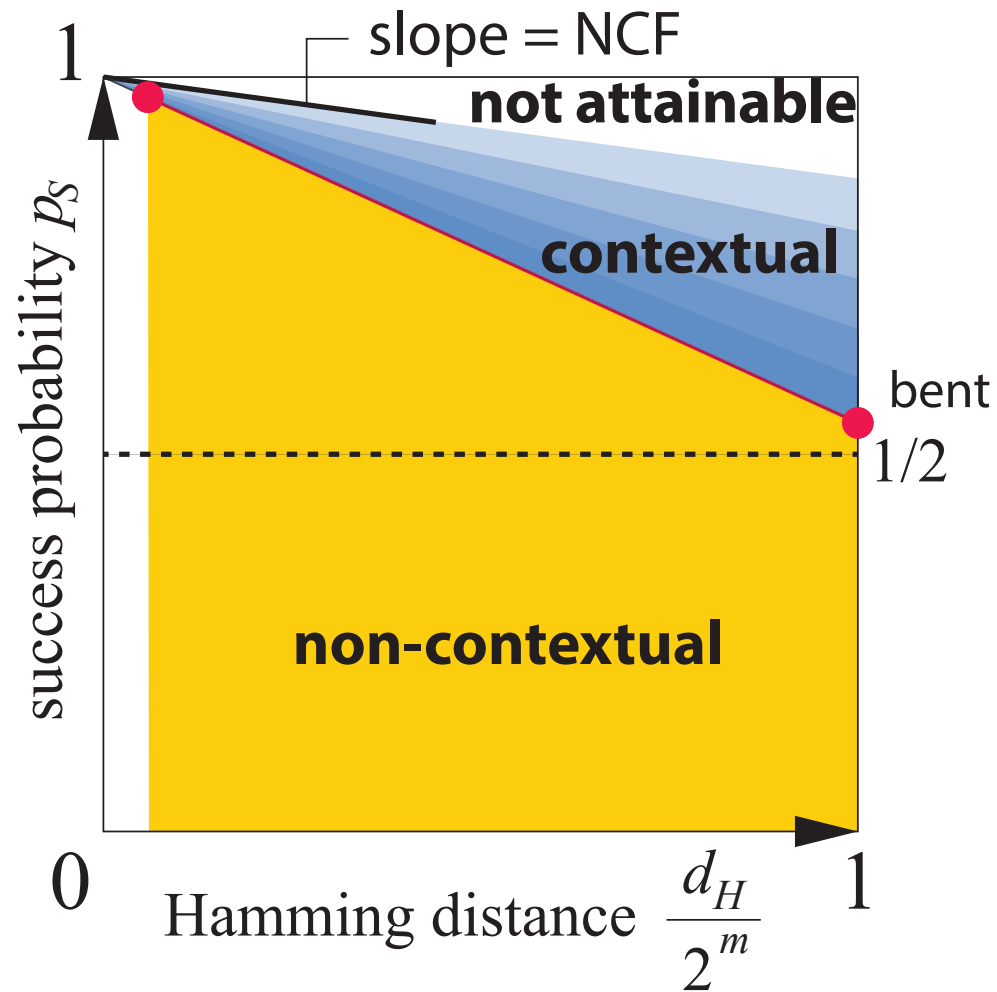
** : S. Abramsky, R.S. Barbosa, S. Mansfield, Phys. Rev. Lett. 119, 050504 (2017).

(i) Sorting out MBQC \longleftrightarrow contextuality



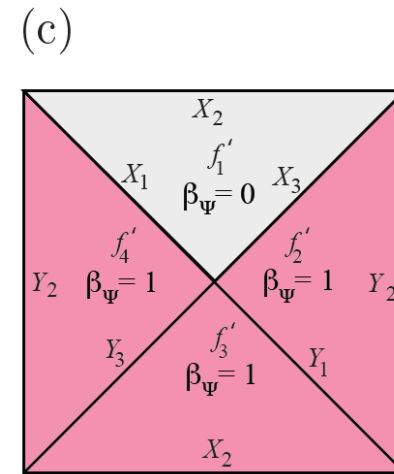
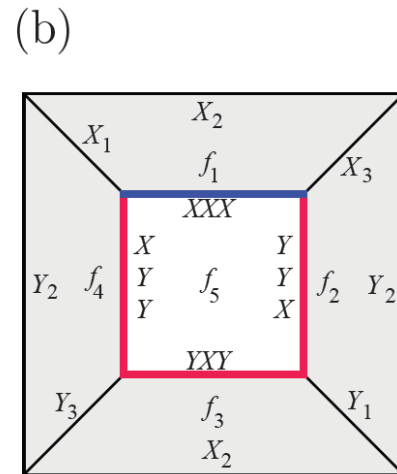
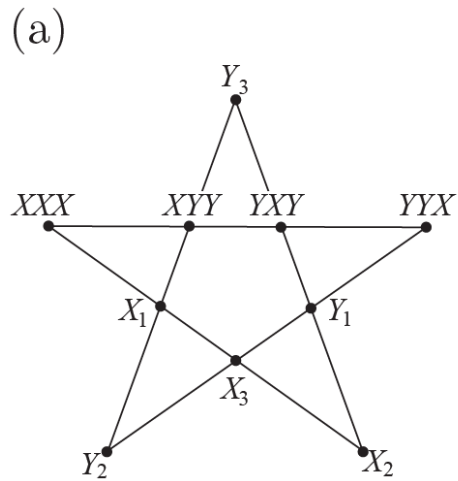
R. Raussendorf, PRA, 022322 (2013).

(i) Sorting out MBQC \longleftrightarrow contextuality



S. Abramsky, R.S. Barbosa, S. Mansfield, PRL 119, 050504 (2017).

(ii) Cohomology \leftrightarrow contextuality'

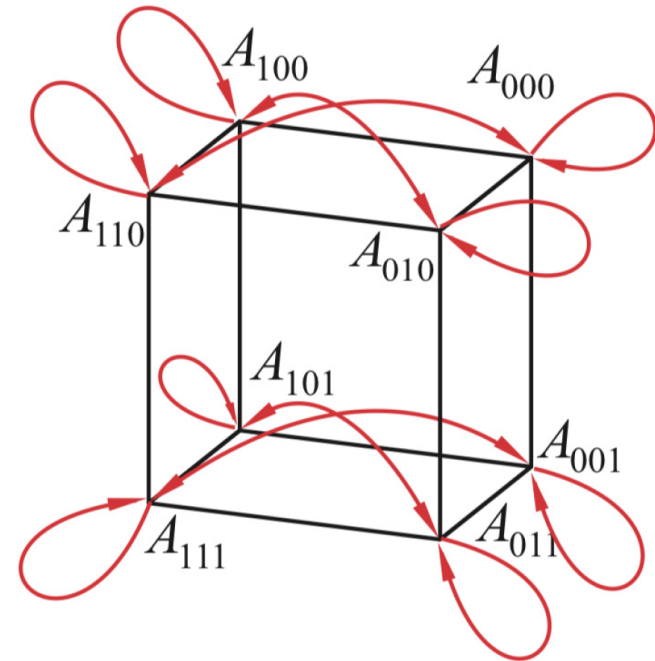


Theorem. An arrangement of observables is contextual if the 2-cocycle class $[\beta] \neq 0$.

C Okay, S Roberts, SD Bartlett, R Raussendorf, *Topological proofs of contextuality in quantum mechanics*, Quant. Inf. Comp. 17, 1135-1166 (2017).

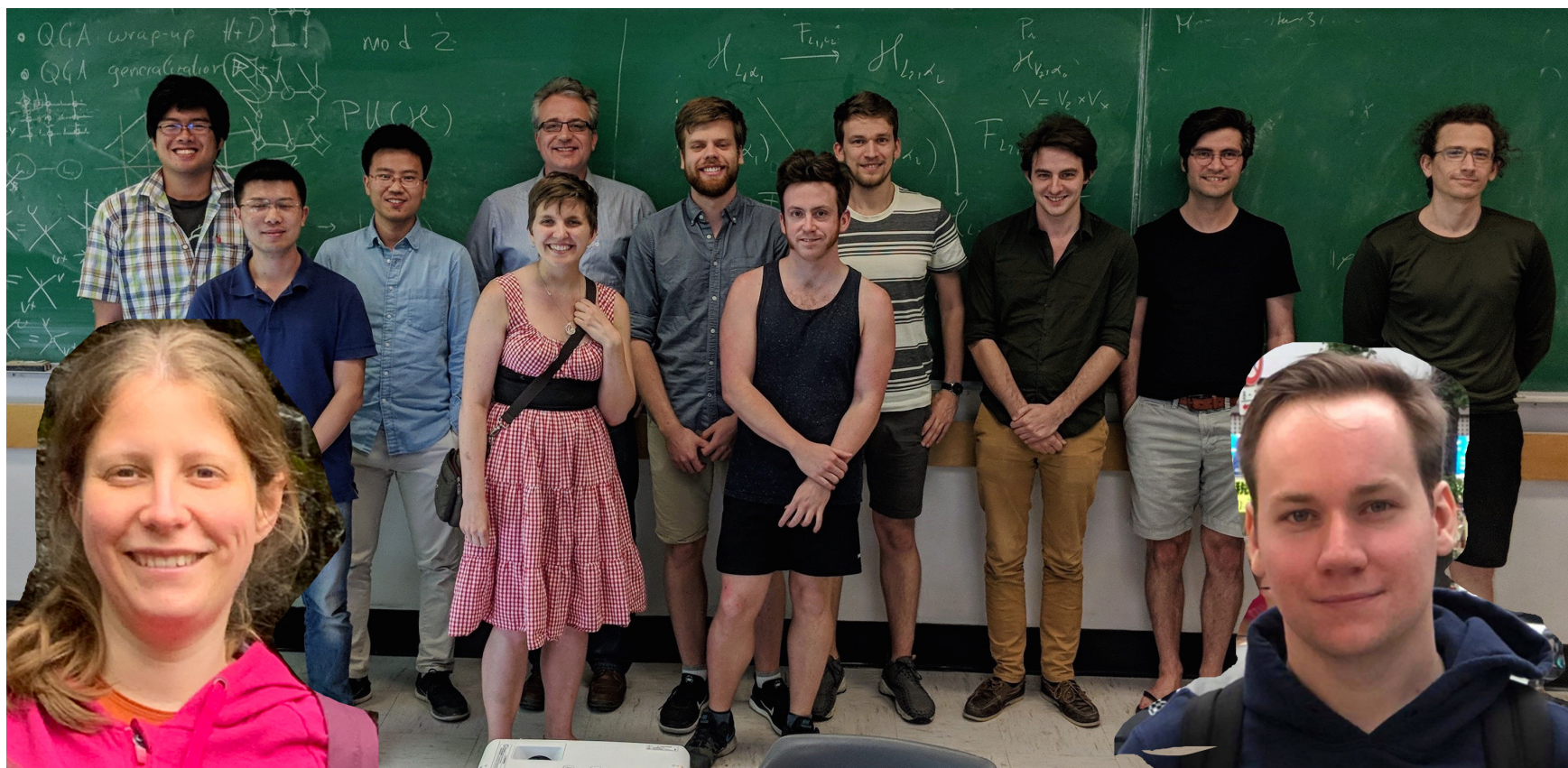
(iii) It's all positive

A counterpoint to the Wigner-negativity-as-quantum-resource body of work:



Theorem. Universal quantum computation can be represented by repeated sampling from probability distributions over finite state space.

M. Zurel, C. Okay, R. Raussendorf, *A hidden variable model for universal quantum computation with magic states on qubits*, PRL 125, 260404 (2020).



Most in May 2018 (ASQC 3 @ UBC)