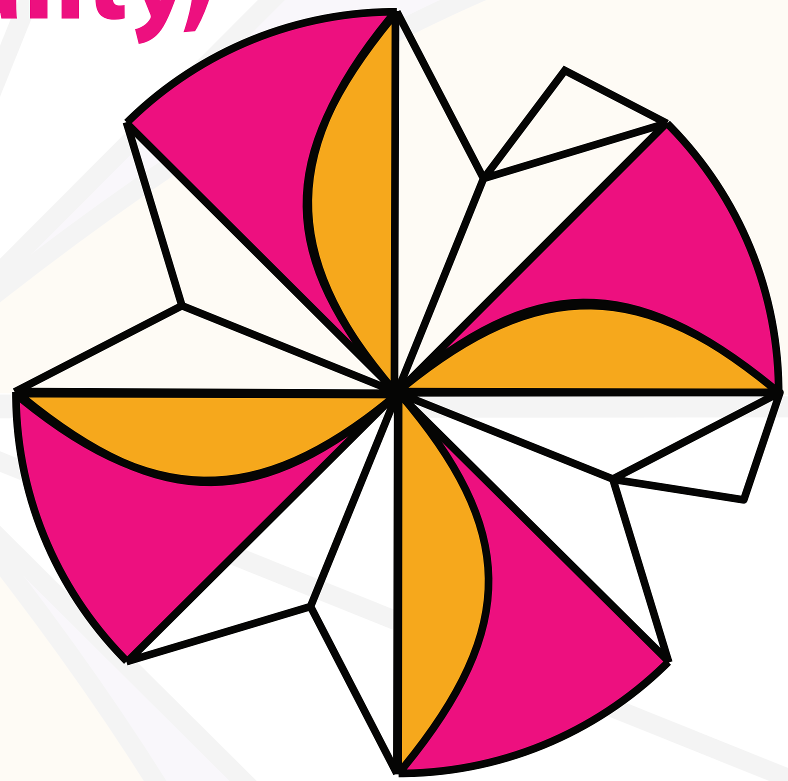
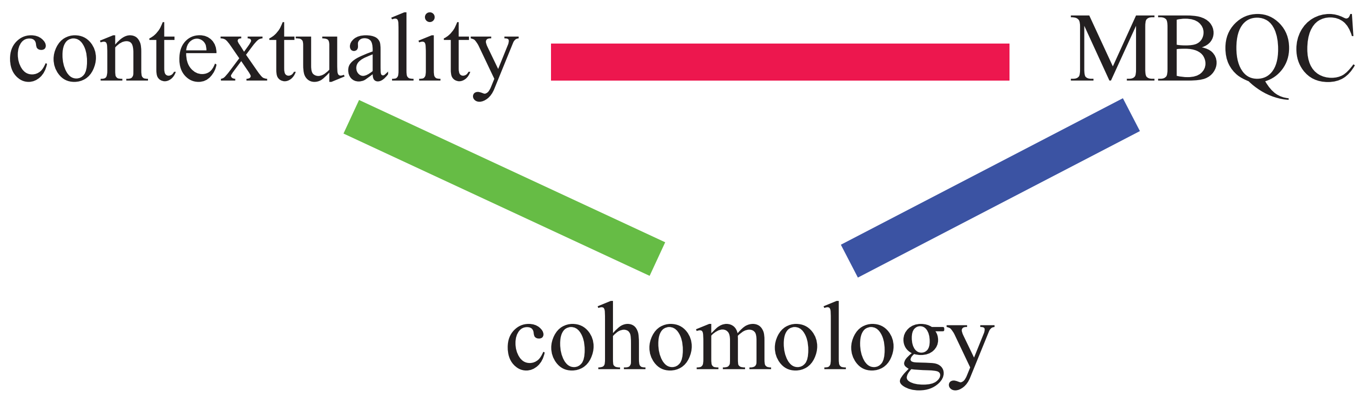
Interwoven paths: my journey through contextuality, cohomology and paradox



Robert Raussendorf, Leibniz Universität Hannover

University College London, September 2023



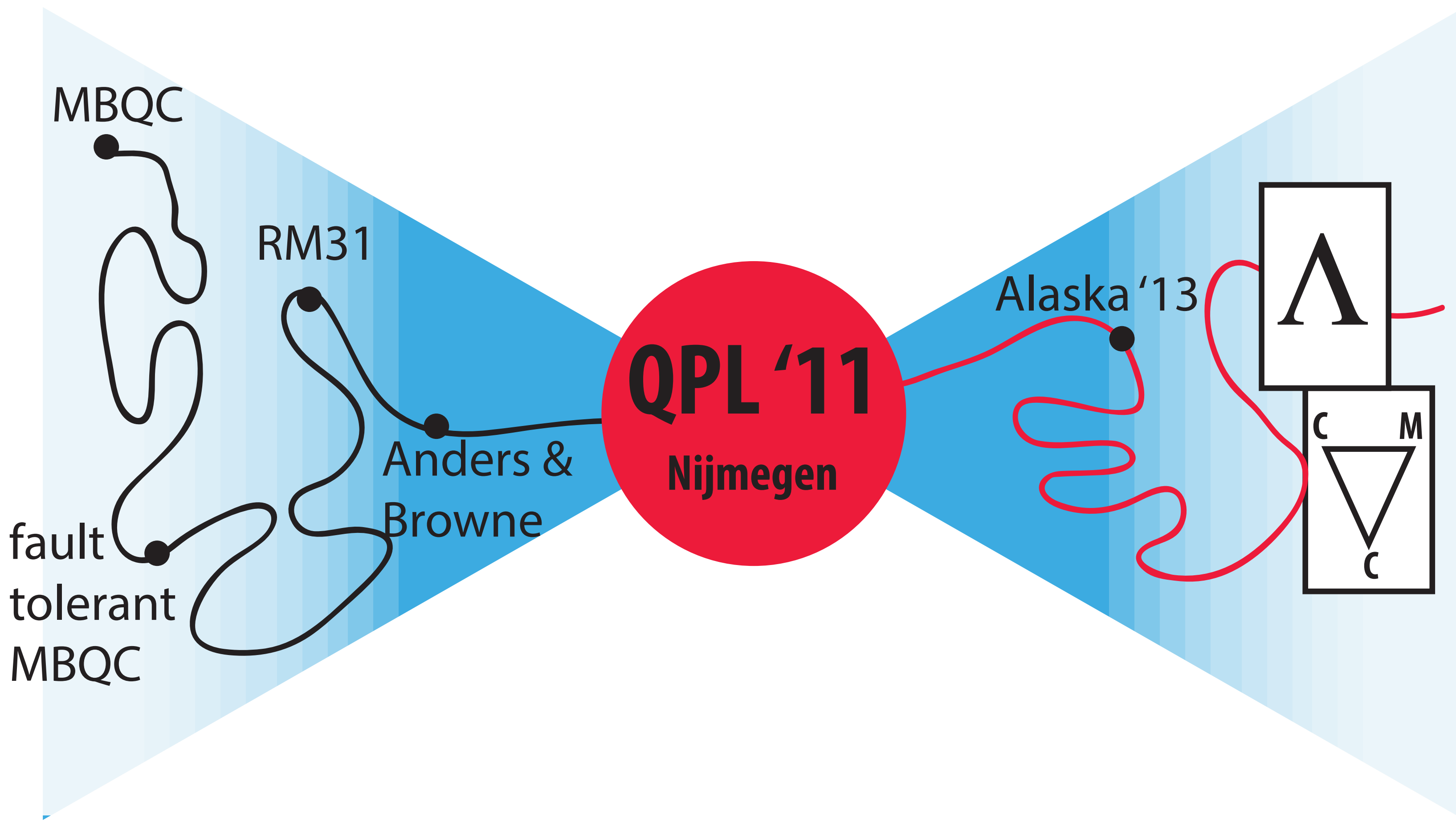
The contextuality – MBQC –cohomology triangle

QPL '11

Nijmegen

6 Travel log

As I learned over the years, the 8th Conference on Quantum Physics and Logic, held in Nijmegen, the Netherlands in November 2011, is remembered fondly by many participants; for all sorts of reasons. Here I'd like to describe my journey towards this conference, how I spiralled out of it, and my thoughts for the future.



6 Travel log

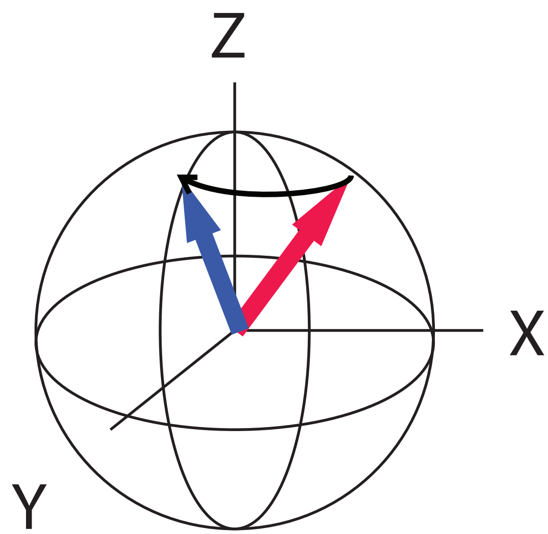
As I learned over the years, the 8th Conference on Quantum Physics and Logic, held in Nijmegen, the Netherlands in November 2011, is remembered fondly by many participants; for all sorts of reasons. Here I'd like to describe my journey towards this conference, how I spiralled out of it, and my thoughts for the future.

1

Measurement-based quantum computation

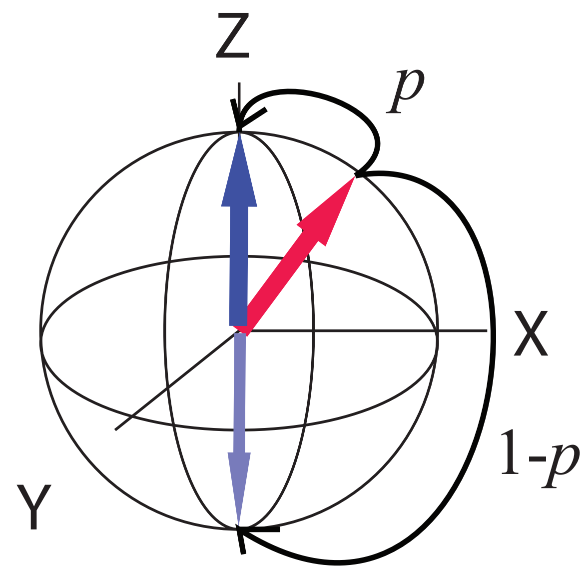
Measurement-based quantum computation

Unitary transformation



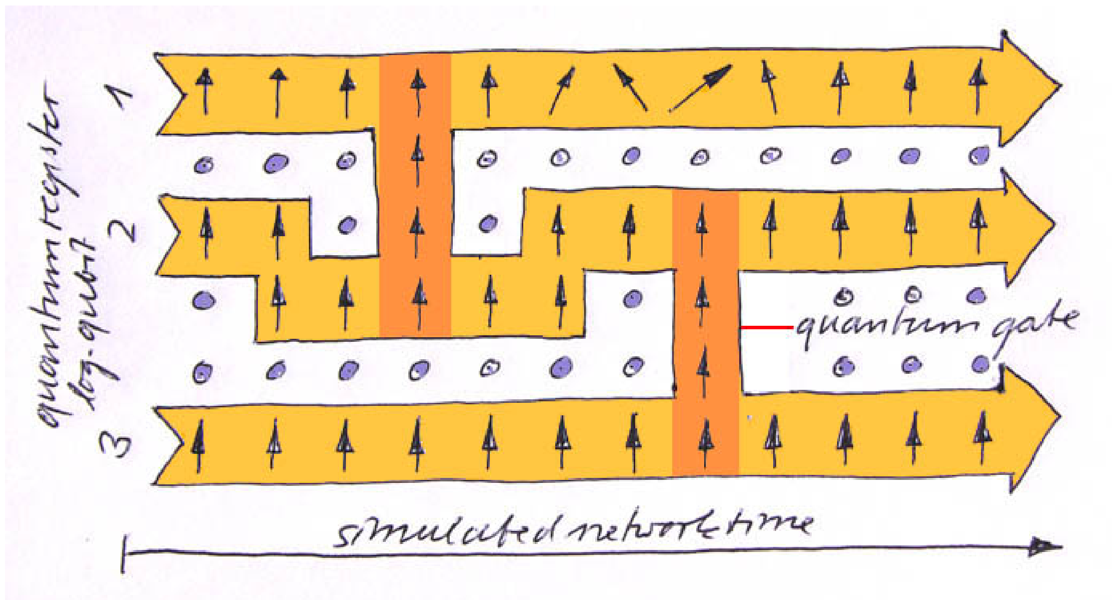
deterministic,
reversible

Projective measurement



probabilistic,
irreversible

Measurement-based quantum computation

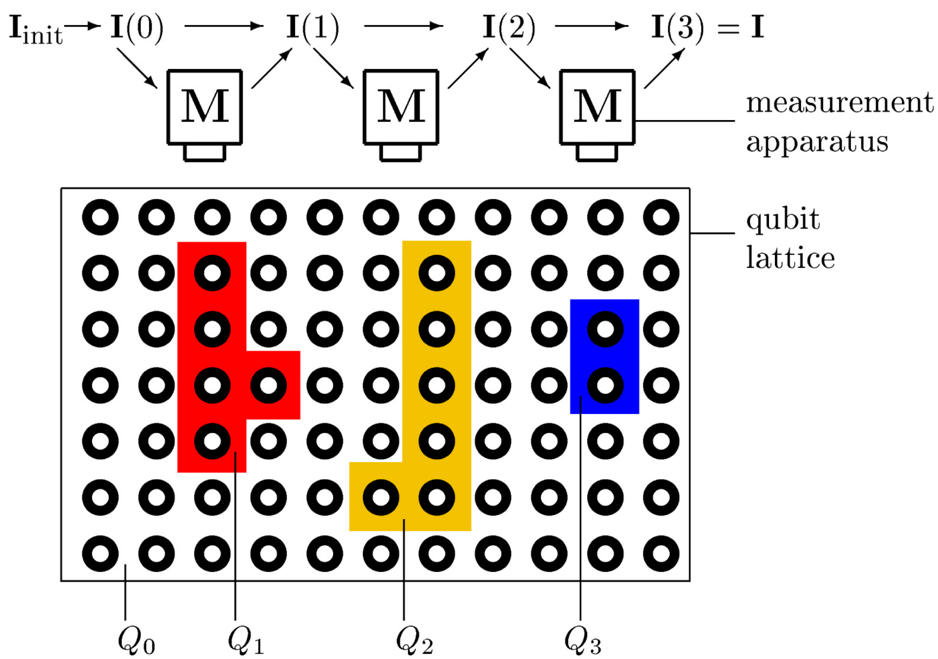


measurement of Z ($\odot$), X ($\uparrow$), $\cos\alpha\, X + \sin\alpha\, Y$ ($\nearrow$)

- Information written onto the resource state, processed and read out by one-qubit measurements only.
- Universal computational resources exist: cluster state, AKLT state.

R. Raussendorf, H.-J. Briegel, Physical Review Letters 86, 5188 (2001).

Measurement-based quantum computation



- The outcome bits of the computations are *correlations* among measurement outcomes.

  Correlations ferreted out by *linear* classical side processing.

R. Raussendorf and H.J. Briegel, *Computational model underlying the one-way quantum computer*, Quant. Inf. Comp. 6, 443 (2002).

2

Fault tolerant measurement-based quantum computation

New Years Card 2004

Progress up to 2023

Fault-tolerant MBQC

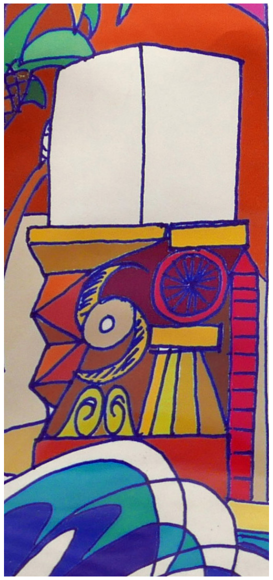
- I expected: Fault-tolerance in MBQC could only be resolved if we understood the non-Pauli correlations in MBQC.

  Solving fault-tolerance for MBQC would combine the interesting with the useful — a goldilocks problem.

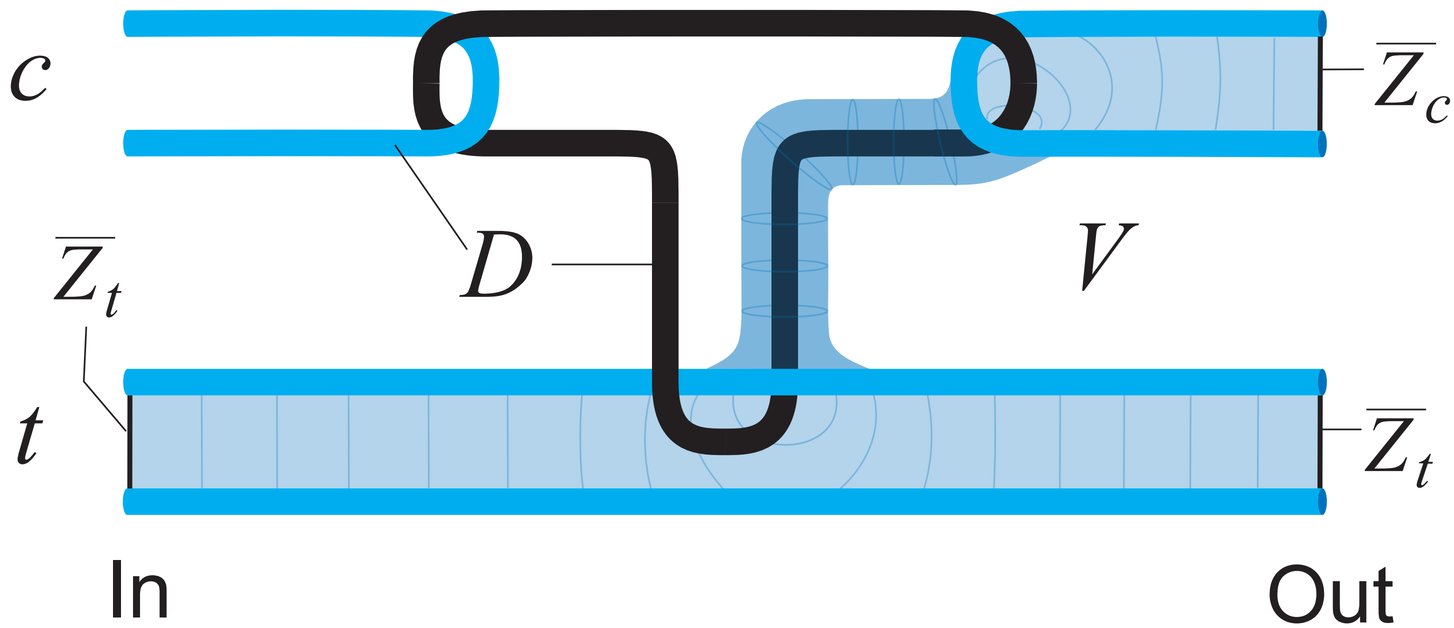
- I anticipated: first construction would be cumbersome, and fail.

- 2005: We solved it!

-



Fault-tolerant MBQC



Topologically protected CNOT gate in 3D cluster states

R. Raussendorf, J. Harrington, K. Goyal, Ann. Phys. (N.Y.) 321, 2242 (2006).

R. Raussendorf and J. Harrington, Phys. Rev. Lett. 98, 190504 (2007).

Fault-tolerant MBQC

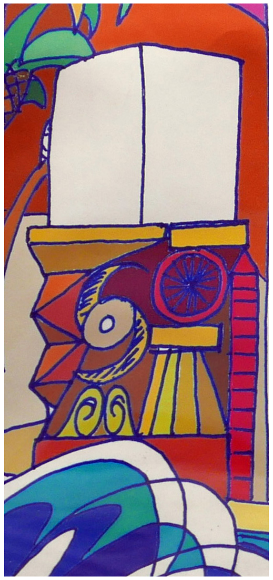
- I expected: Fault-tolerance in MBQC could only be resolved if we understood the non-Pauli correlations in MBQC.

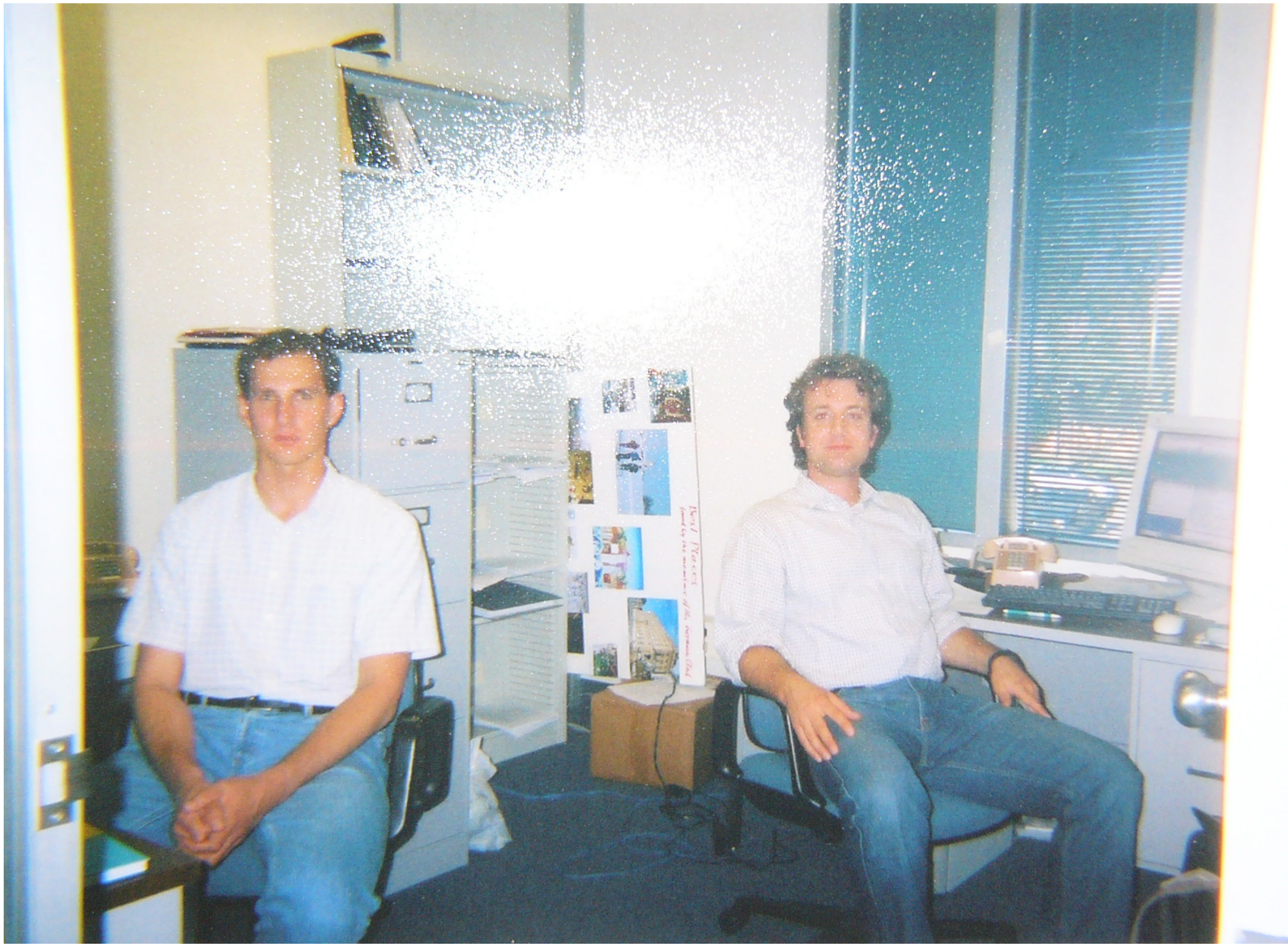
  Solving fault-tolerance for MBQC would combine the interesting with the useful — a goldilocks problem.

- I anticipated: first construction would be cumbersome, and fail.

- 2005: We solved it!

- *The non-Pauli correlations did not need to be understood to solve fault-tolerance for MBQC.*





Sergey Bravyi and I shared an office at IQI

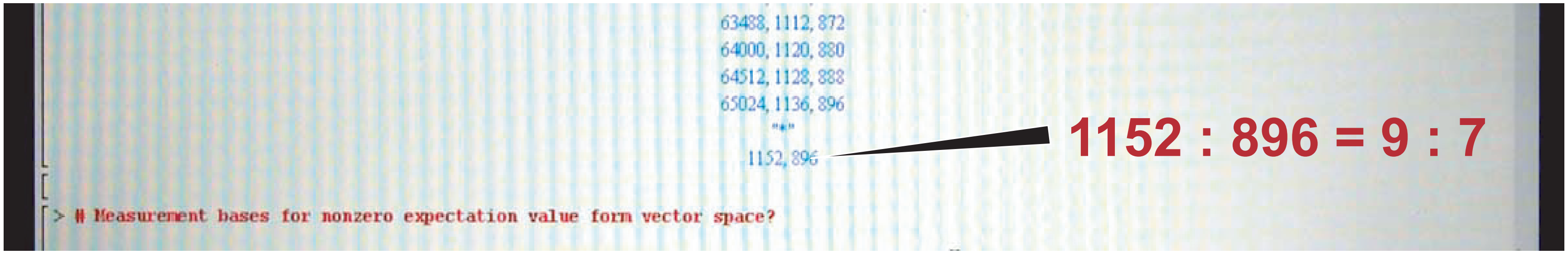
Sergey $\longrightarrow$ magic state distillation $\longrightarrow$ Reed-Muller codes

3

RM31 *

*: for Reed-Muller

Why not use RM code states for MBQC?



Reed-Muller code states provide MBQC resource states for

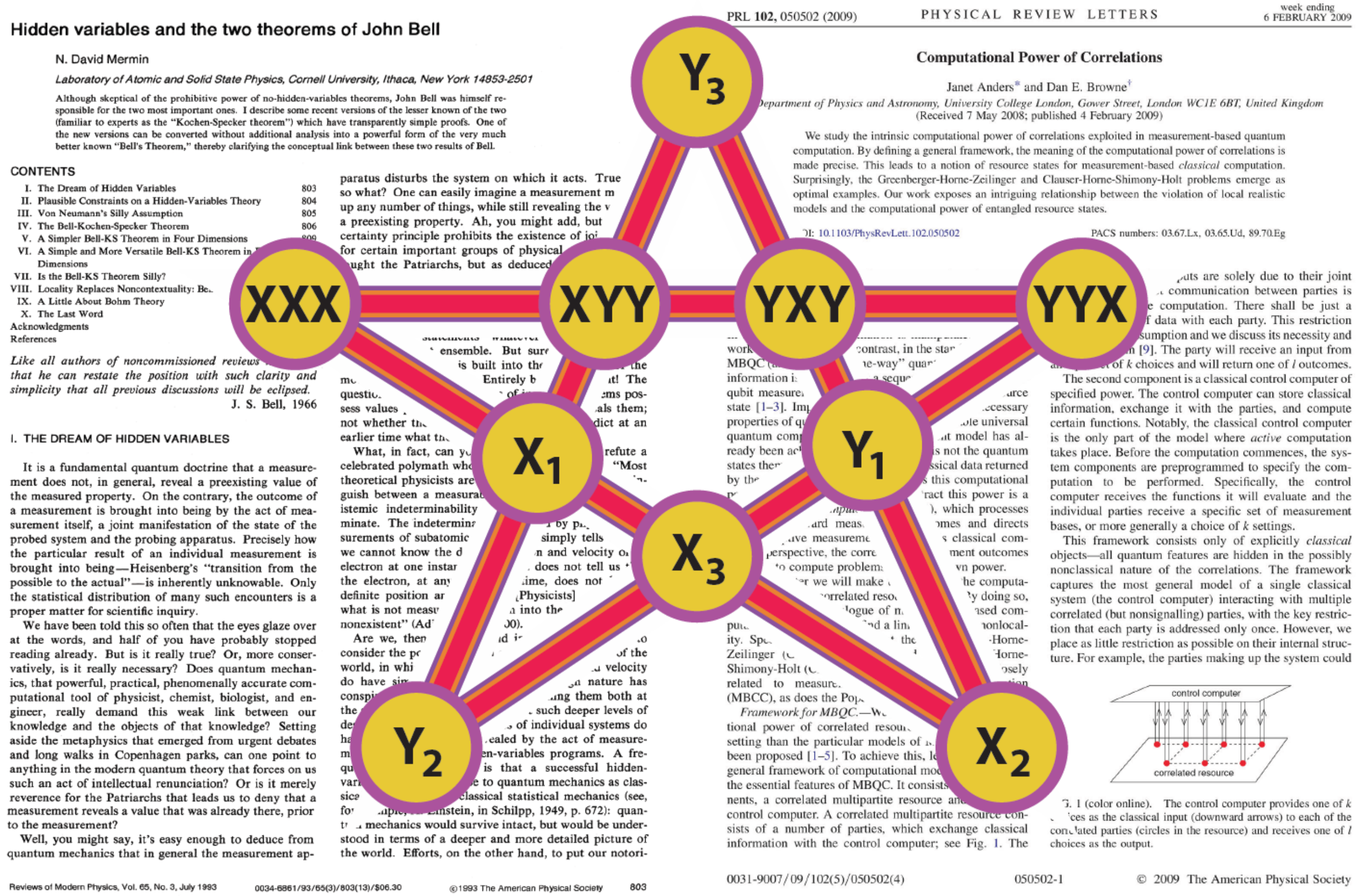
- Deterministically computing a non-linear Boolean function,
- While obeying the linear classical side processing relations of MBQC, and
- Being non-Clifford.

All three criteria satisfied for 31 qubits.

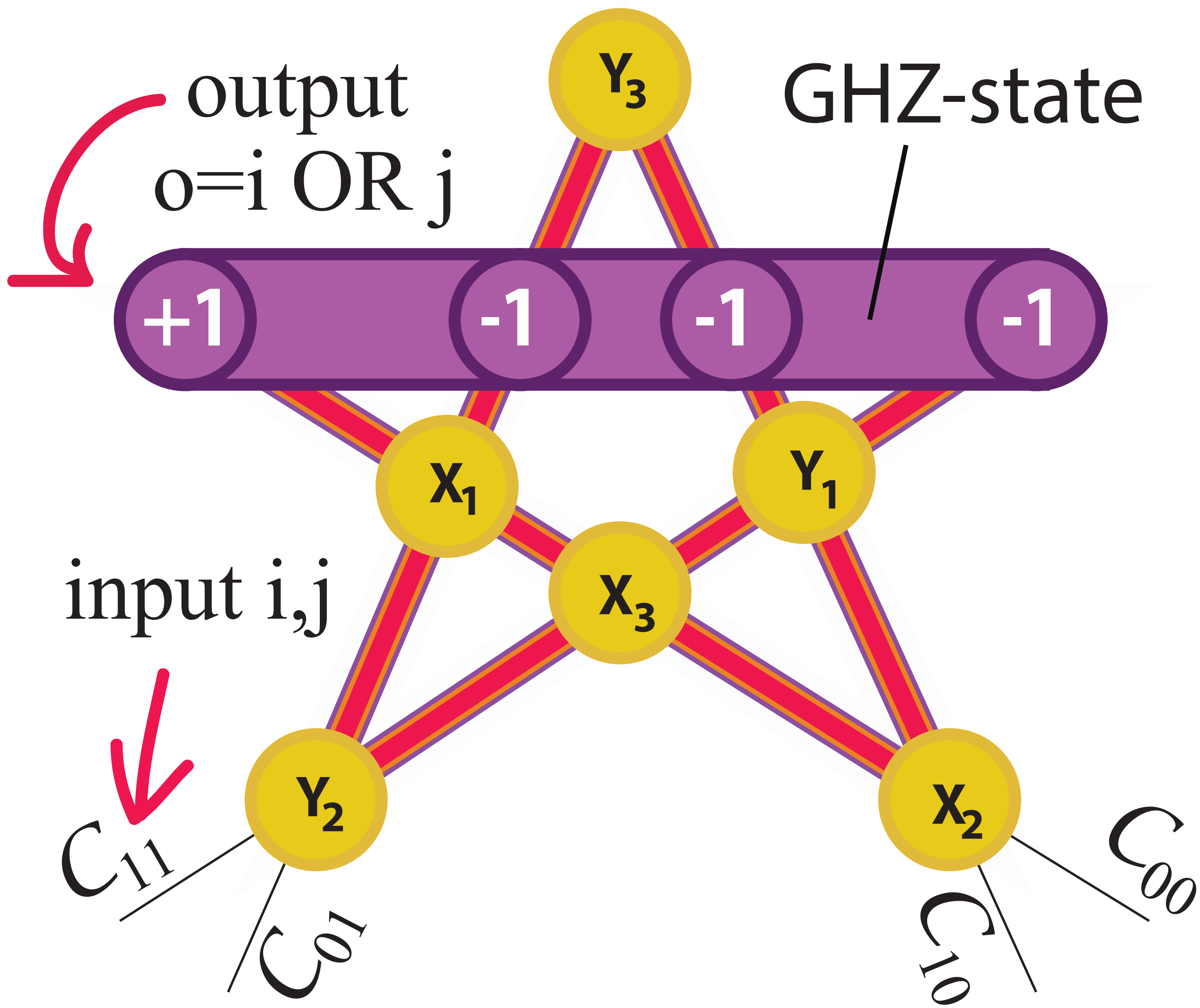
(These are toy computations)

4

Contextuality in MBQC: Anders & Browne



- Mermin's star, a contextuality proof on 3 qubits, can be repurposed as an MBQC!


output
o=i OR j
GHZ-state
Y_3
+1
-1
-1
-1
X_1
Y_1
X_3
input i,j
Y_2
X_2
C_{11}
C_{01}
C_{10}
C_{00}

QPL 2011

Contextuality and Cohomology:
Abramsky, Barbosa, Mansfield

The Cohomology of Non-Locality and Contextuality

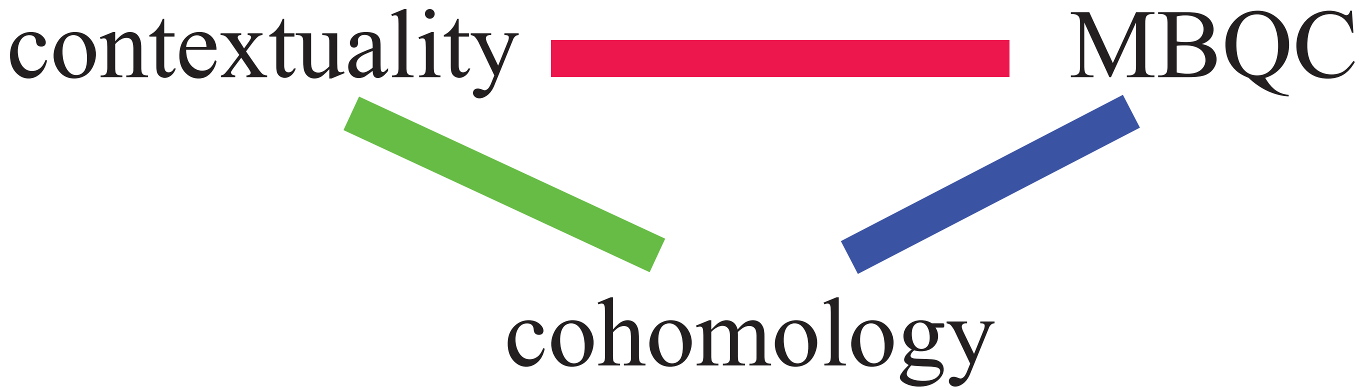
Samson Abramsky Shane Mansfield Rui Soares Barbosa

Department of Computer Science
University of Oxford

{samson.abramsky,shane.mansfield,rui.soaresbarbosa}@cs.ox.ac.uk

In a previous paper with Adam Brandenburger, we used sheaf theory to analyze the structure of non-locality and contextuality. Moreover, … is formulation, we showed that the phenomena of non-locality and contextu… precisely in terms … to the existence of global sections.

Seemingly in close reach after QPL ’11



The contextuality – MBQC –cohomology triangle

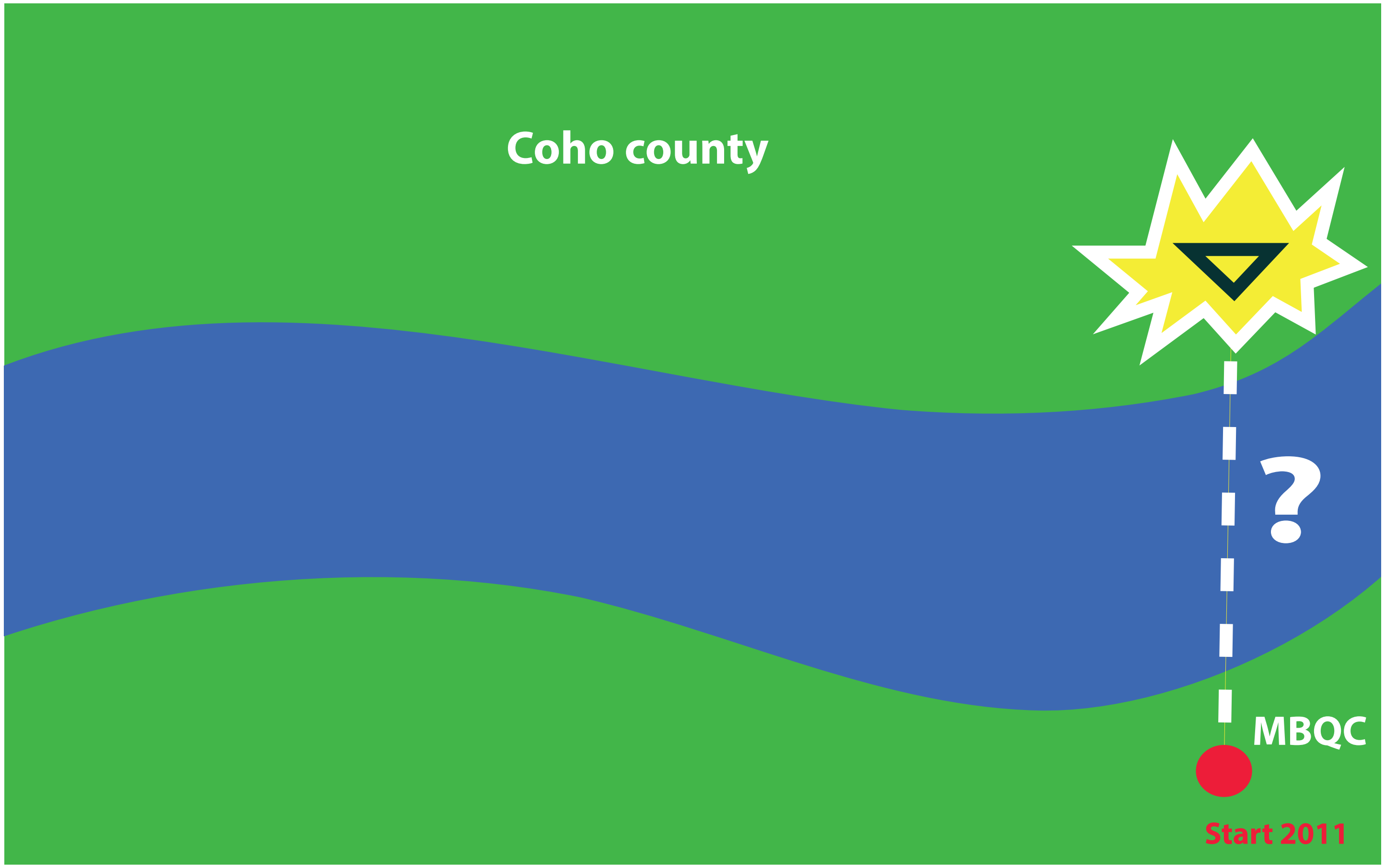
Partially established (temporally flat MBQCs only)

Inspirations:

Anders and Browne, *Computational Power of Correlations*, PRL 102 (2009),

Abramsky, Barbosa, Mansfield, *Cohomology of contextuality*, arXiv:1111.3620.

But ...

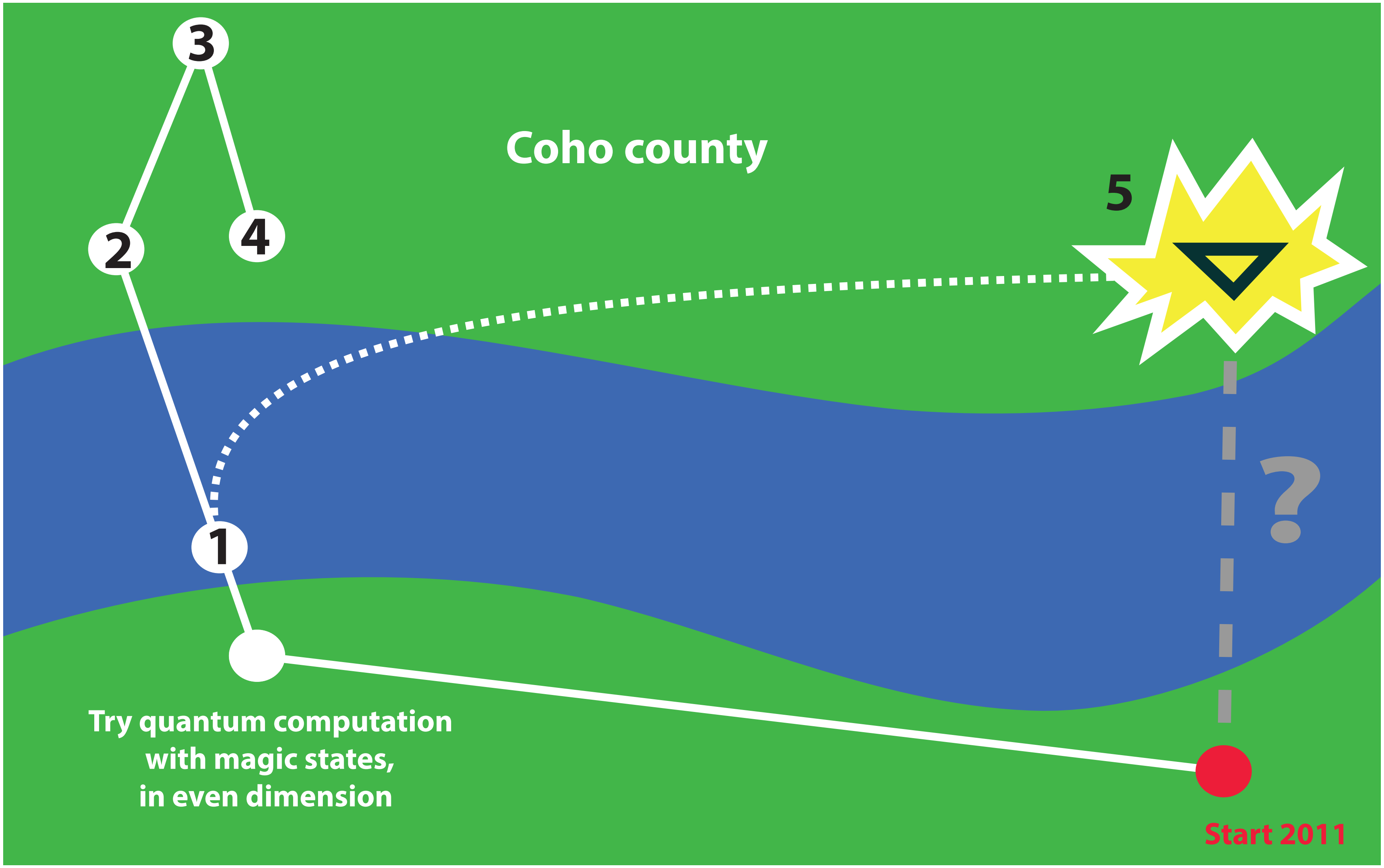


Unlocking the triangle proved to be harder than thought.

6

Alaska,
Summer 2013

Joseph Emerson and Stephen Bartlett, July 2013

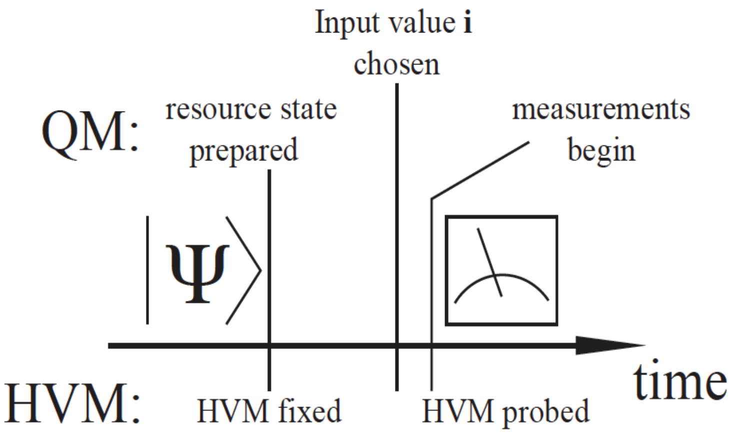


3
2
4
Coho county
5
1
?
Try quantum computation
with magic states,
in even dimension
Start 2011

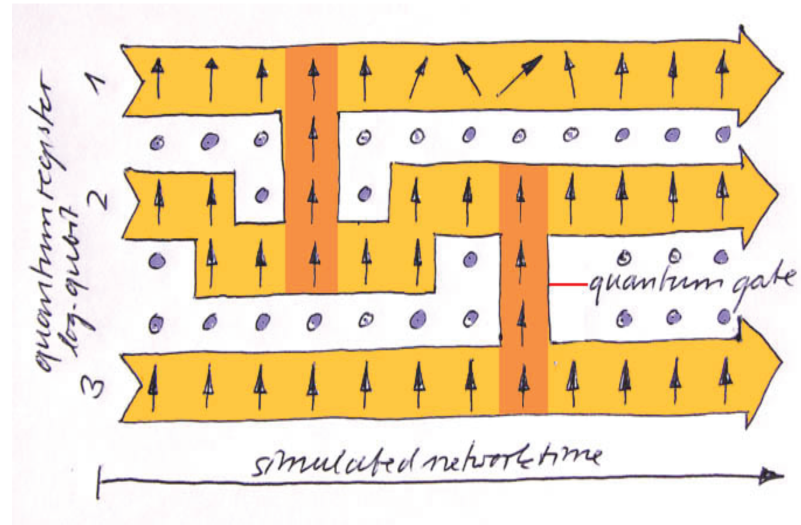
7

Interaction Picture

(i) Sorting out MBQC $\longleftrightarrow$ contextuality



?
=



Theorem 1.* An MBQC evaluating a nonlinear Boolean function $o : (\mathbb{Z}_2)^m \longrightarrow \mathbb{Z}_2$ deterministically is contextual.

*: R. Raussendorf, Phys. Rev. A, 022322 (2013).

(i) Sorting out MBQC $\longleftrightarrow$ contextuality

Theorem 2.* Be $\mathcal{M}$ an MBQC evaluating a nonlinear Boolean function $o : (\mathbb{Z}_2)^m \longrightarrow \mathbb{Z}_2$ with average success probability p_S. Then, $\mathcal{M}$ is contextual if $p_S > 1 - 1/2^m$, and, for bent functions, if $p_S > 1/2 + 1/2^{m/2+1}$.

Theorem 3.** Be $\mathcal{M}$ an MBQC evaluating a nonlinear Boolean function $o : (\mathbb{Z}_2)^m \longrightarrow \mathbb{Z}_2^l$ with average success probability p_S. Then,

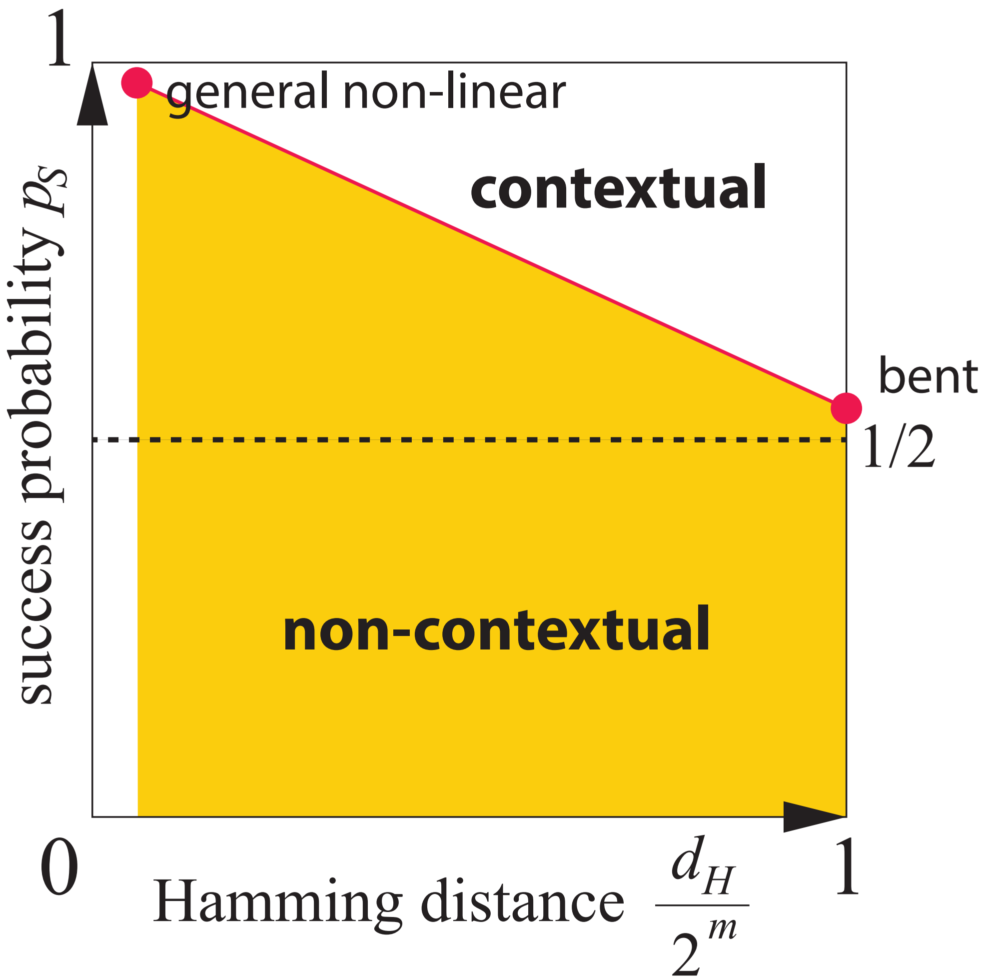
$$p_S \leq 1 - \mathsf{NCF}\frac{d_H(o)}{2^m}.$$

Therein, $d_H(o)$ is the Hamming distance from the closest linear function.

*: R. Raussendorf, Phys. Rev. A, 022322 (2013).

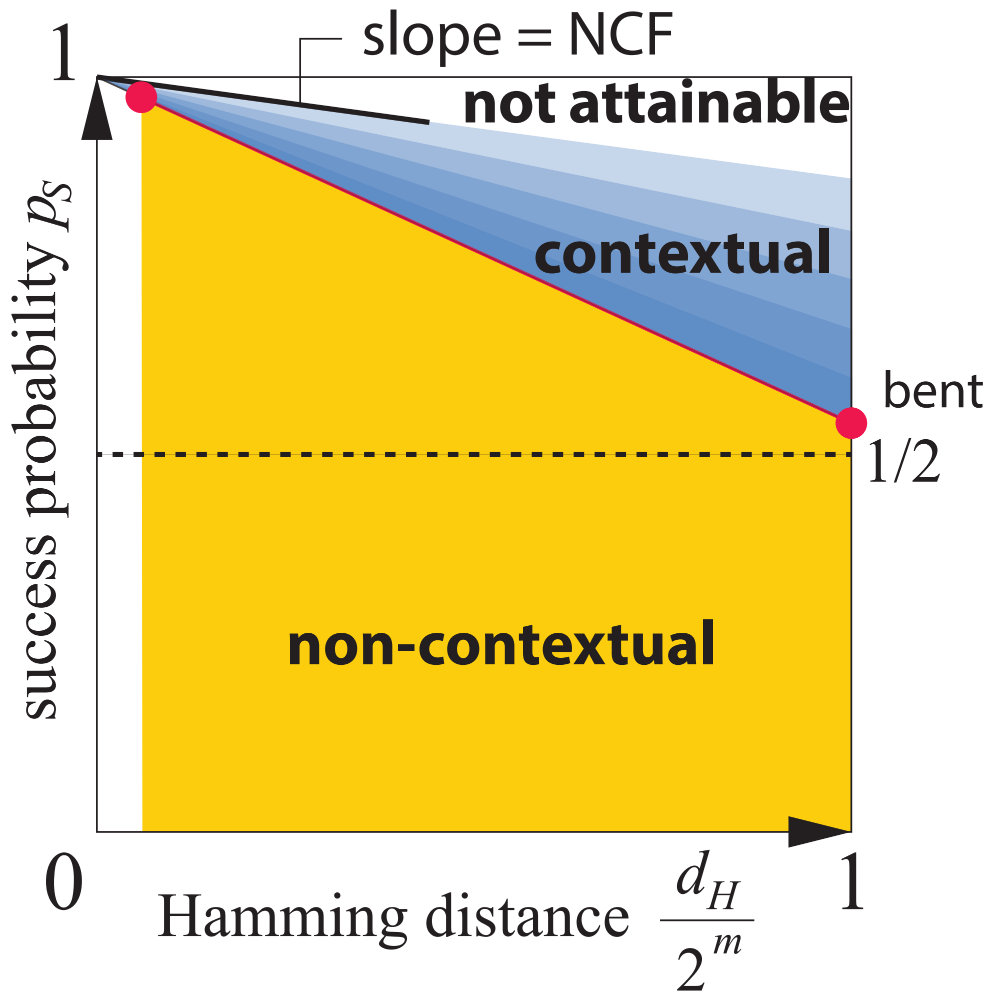
**: S. Abramsky, R.S. Barbosa, S. Mansfield, Phys. Rev. Lett. 119, 050504 (2017).

(i) Sorting out MBQC $\longleftrightarrow$ contextuality



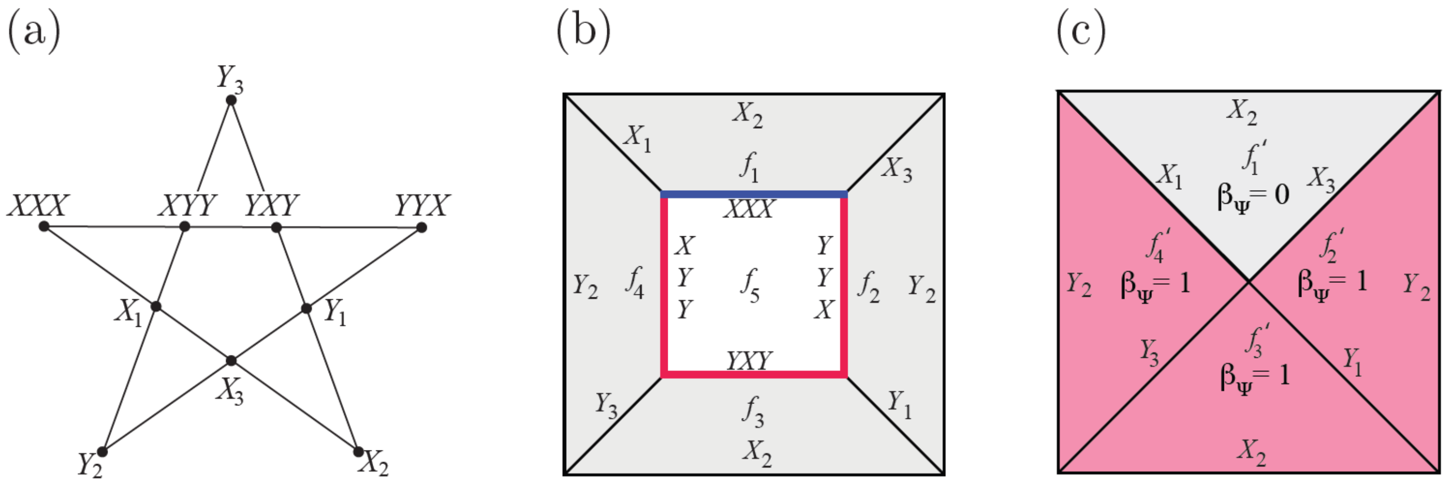
R. Raussendorf, PRA, 022322 (2013).

(i) Sorting out MBQC $\longleftrightarrow$ contextuality



S. Abramsky, R.S. Barbosa, S. Mansfield, PRL 119, 050504 (2017).

(ii) Cohomology ↔ contextuality'

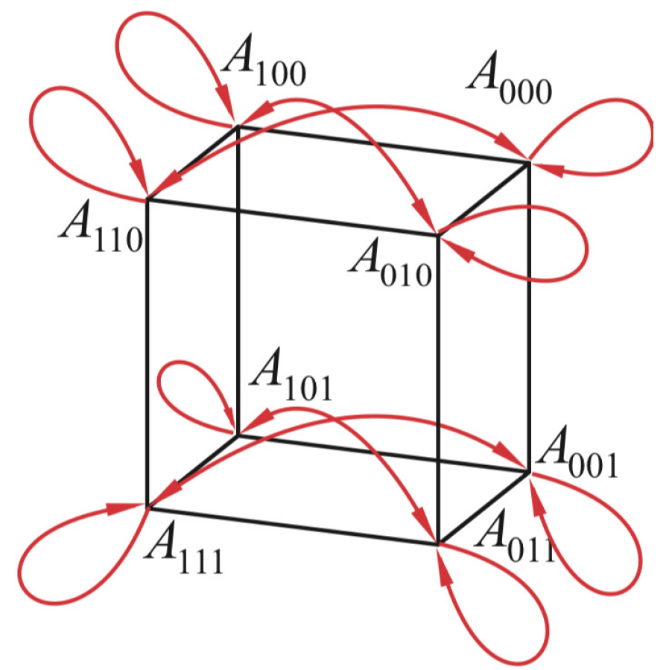


Theorem. An arrangement of observables is contextual if the 2-cocycle class $[\beta] \neq 0$.

C Okay, S Roberts, SD Bartlett, R Raussendorf, *Topological proofs of contextuality in quantum mechanics*, Quant. Inf. Comp. 17, 1135-1166 (2017).

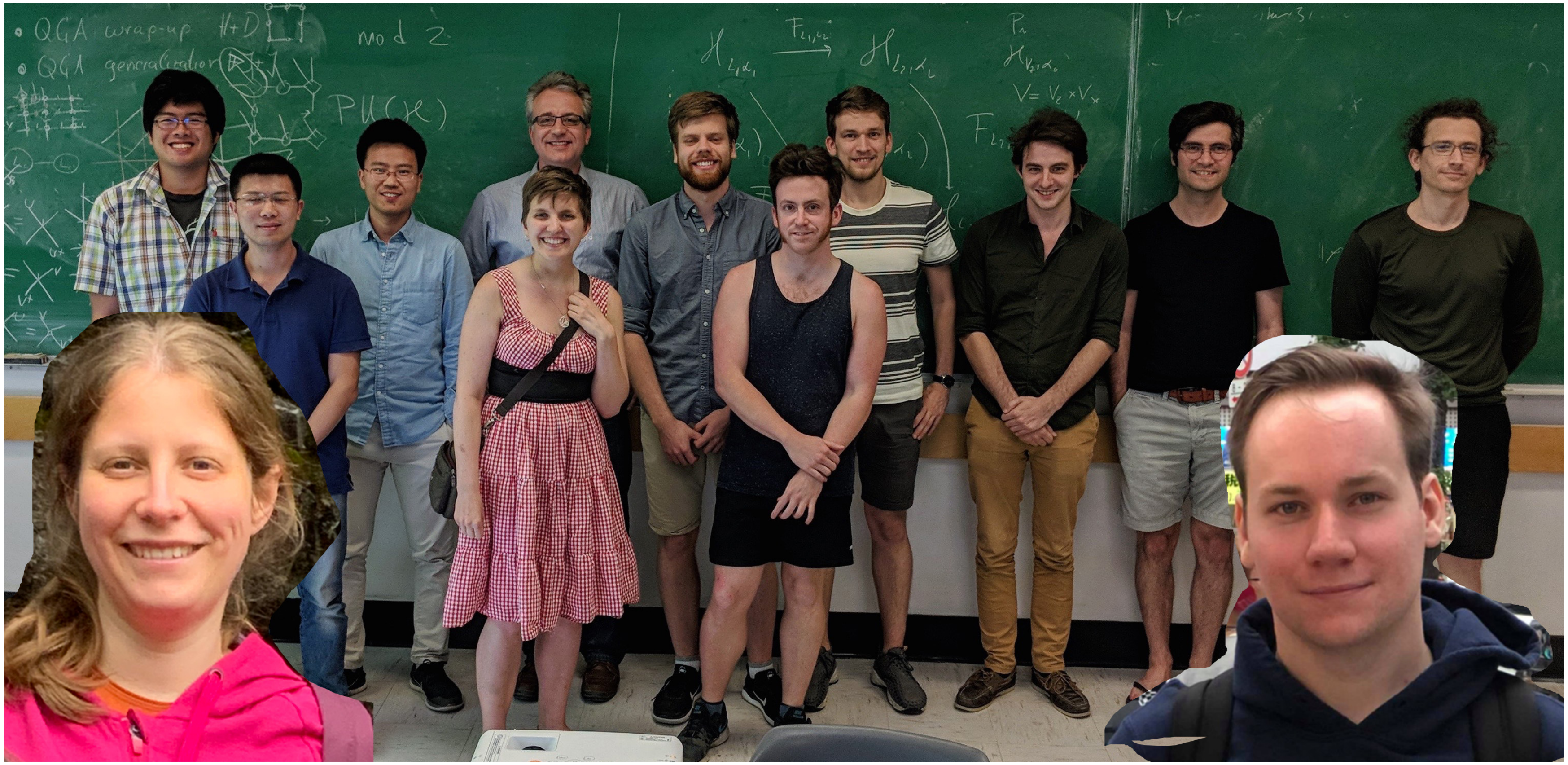
(iii) It's all positive

A counterpoint to the Wigner-negativity-as-quantum-resource body of work:



Theorem. Universal quantum computation can be represented by repeated sampling from probability distributions over finite state space.

M. Zurel, C. Okay, R. Raussendorf, *A hidden variable model for universal quantum computation with magic states on qubits*, PRL 125, 260404 (2020).



Most in May 2018 (ASQC 3 @ UBC)